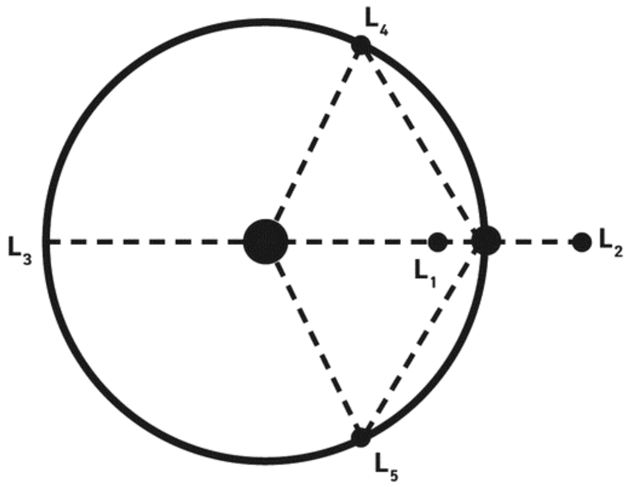
Moscow Aviation Institute
Research Institute of Applied Mechanics and Electrodynamics

Transit trajectories of ballistic capture near libration points for low-energy transfers.

Alexey V. Ivanyukhin

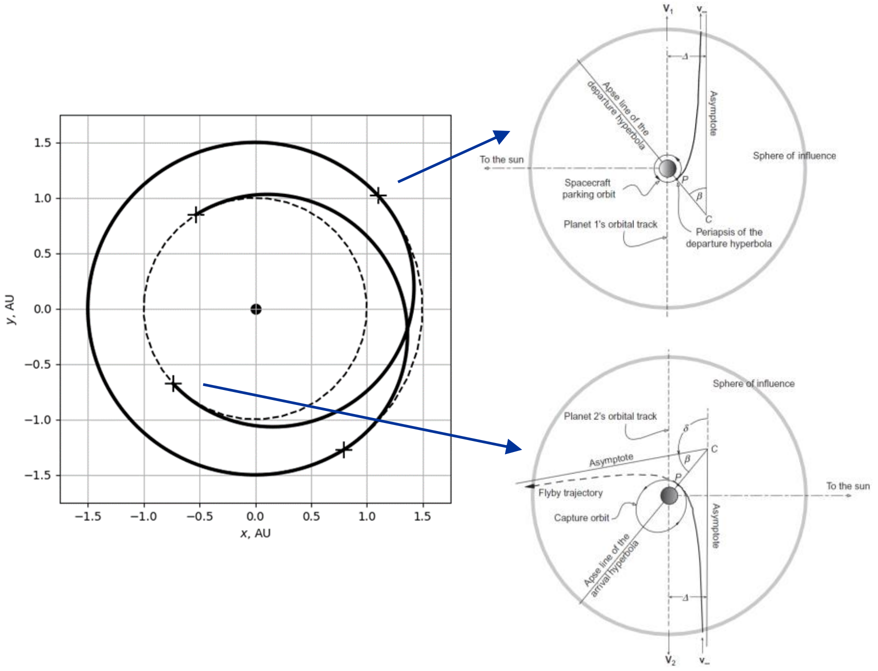


Analytical Methods of Celestial Mechanics 2024
August 19-24, 2024
Euler International Mathematical Institute, St. Petersburg, RUSSIA

Motivation for designing low-energy transfer

Energy change
(for CR3BP, the Jacobi constant) → **Transfer near the libration point**

Less change of the Jacobi constant
≈ Less fuel costs → **Low-thrust**



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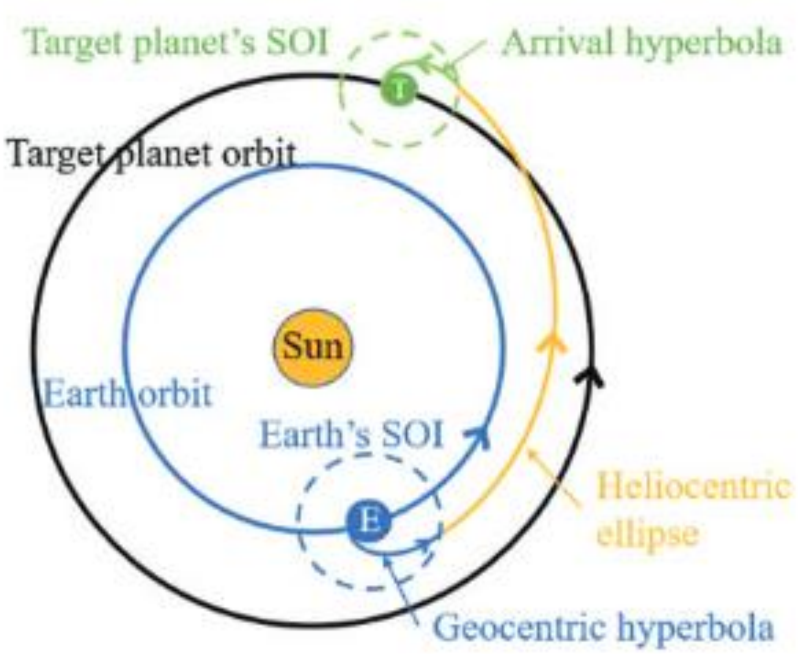

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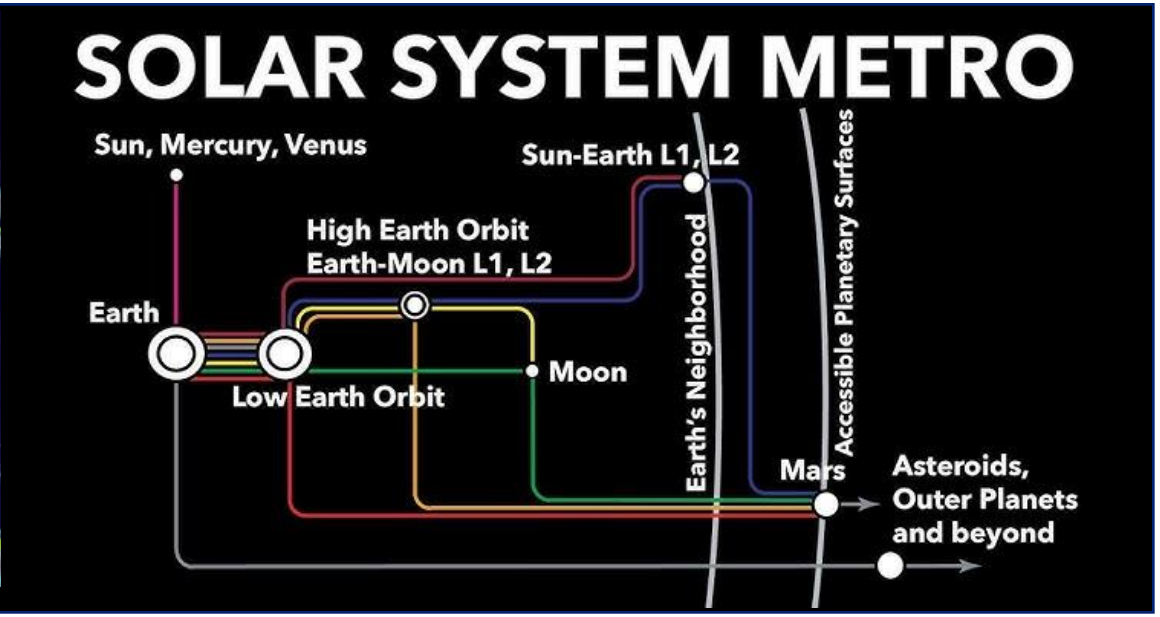
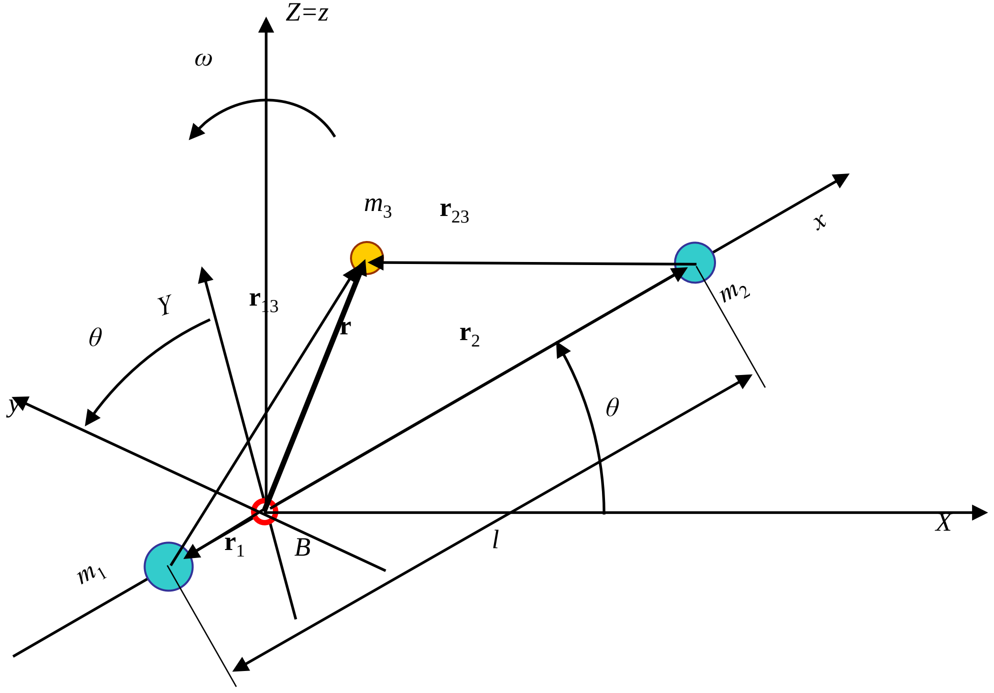


“InterPlanetary Superhighway”, Dynamical systems, the three-body problem and space mission design
(Koon W. S., Lo M. W., Marsden J. E., Ross S. D. - 2022)

The circular restricted three-body problem (CR3BP)



$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} - y \\ \dot{y} + x \\ \dot{z} \end{pmatrix}$$

Massive bodies: $m_3 << m_1 + m_2, \quad m_2 < m_1,$
m_1, m_2 they move along Keplerian (circular) orbits relative to the barycenter;

Movement of body m_3:

- in a inertial (XYZ, cartesian) system:

$$\frac{d^2\mathbf{r}}{dt^2} = -(1-\mu)\frac{\mathbf{r}_{13}}{r_{13}^3} - \mu\frac{\mathbf{r}_{23}}{r_{23}^3};$$

- in a rotating (xyz, synodic) system:

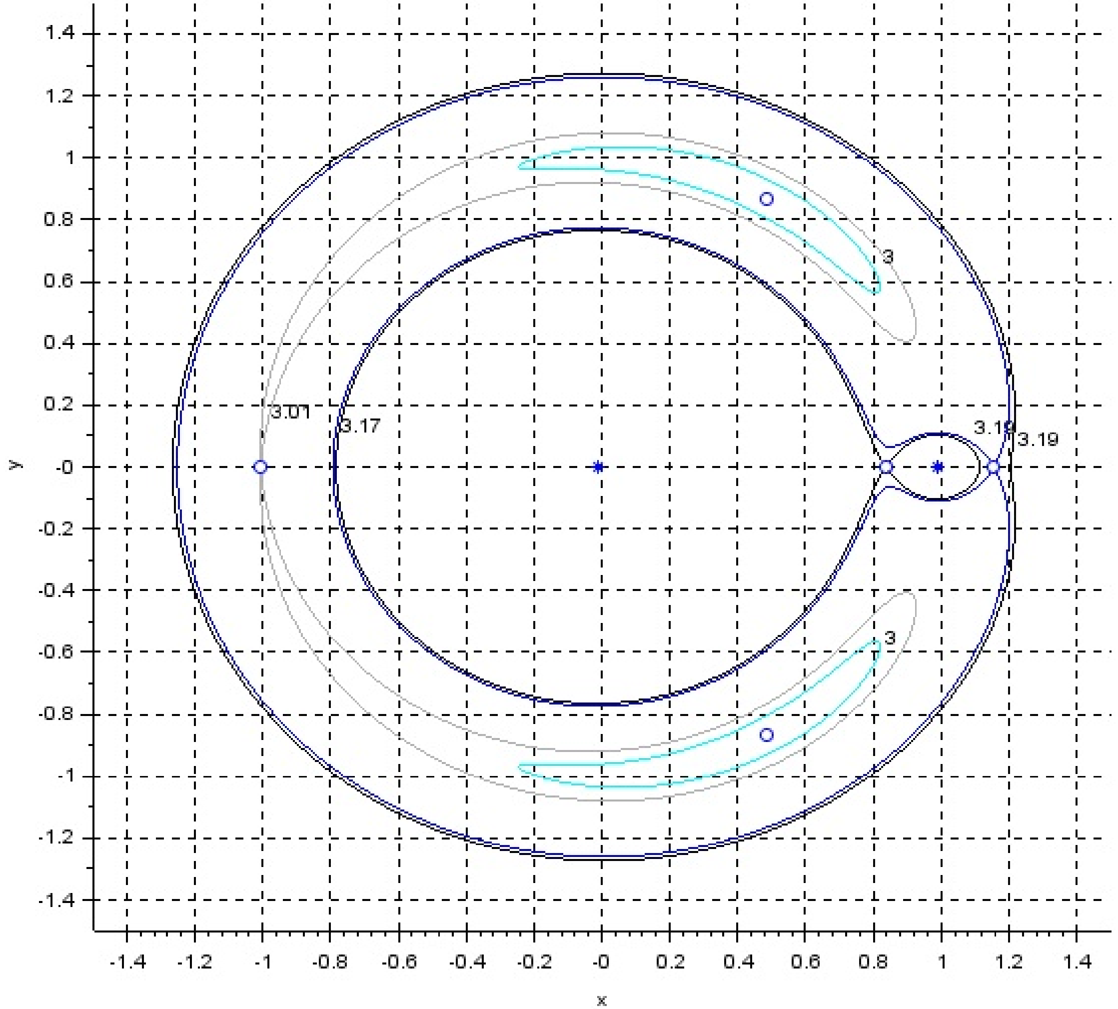
$$\begin{cases} \ddot{x} - 2\dot{y} = U_x, \\ \ddot{y} + 2\dot{x} = U_y, \\ \ddot{z} = U_z, \end{cases} \quad U = \frac{x^2 + y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2};$$

Let $\mu_1 + \mu_2 = 1, \quad l = 1, \quad \mu = \dfrac{m_2}{m_1 + m_2} \le 0.5,$

Then $\omega = \sqrt{\dfrac{\mu_1 + \mu_2}{l^3}} = 1, \quad \theta = \omega t.$

Jacobi integral: $C_J = 2U(\mathbf{r}) - \mathbf{v}^2.$

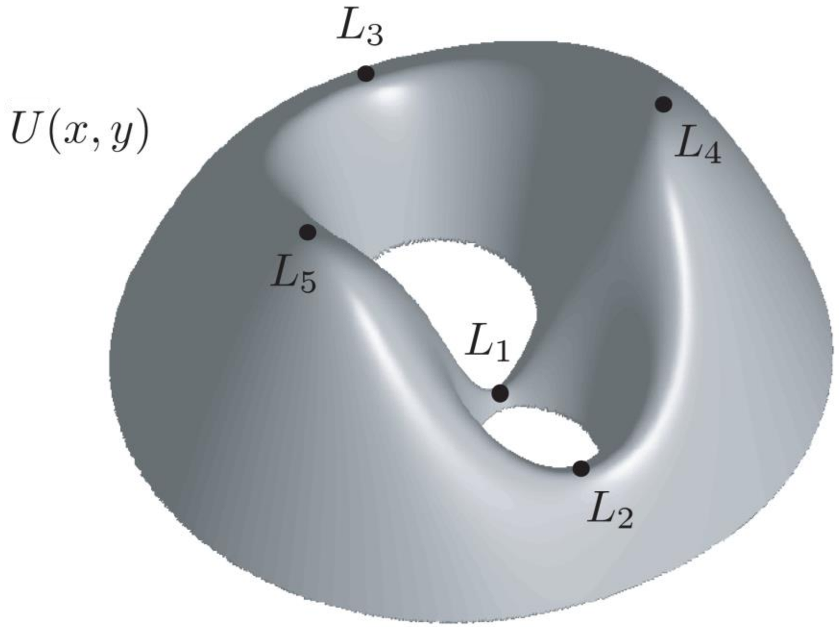
CR3BP: Libration points and Jacobi integral



Jacobi integral (Earth – Moon)

Libration points are stationary solutions of differential equations in the **rotating** system:

$$\dot{\mathbf{r}} = \dot{\mathbf{v}} = 0$$



The effective potential in the plane containing the orbit

CR3BP: $\Delta \mathbf{v}_{\min}$

Jacobi integral: $C_J(\mathbf{r},\mathbf{v}) = 2U(\mathbf{r}) - \mathbf{v}^2.$

$$\Delta C_J(\mathbf{r},\mathbf{v}) = C_J(\mathbf{r},\mathbf{v}+\Delta\mathbf{v}) - C_J(\mathbf{r},\mathbf{v}) = -\Delta\mathbf{v}^T\Delta\mathbf{v} - 2\mathbf{v}^T\Delta\mathbf{v}$$

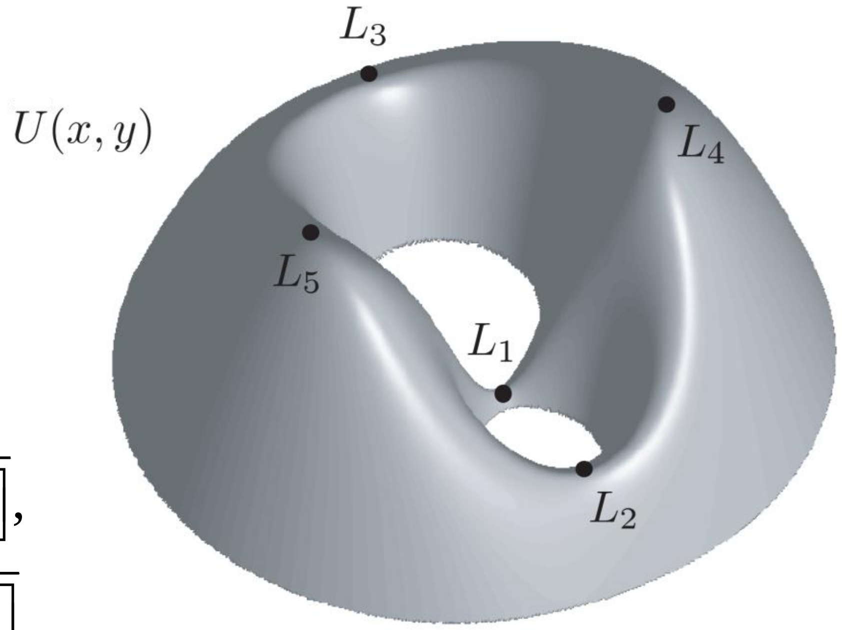
$$\Rightarrow \quad \mathbf{v} \parallel \Delta\mathbf{v}_{\min},$$

$$\Delta C_J(\mathbf{r}_{L1},\mathbf{v}_{L1}) \le C_J(\mathbf{r}+\Delta\mathbf{r},\mathbf{v}+\Delta\mathbf{v})$$

$$\Rightarrow \quad \Delta\mathbf{v}_{\min} = \Delta\mathbf{v}_{1\min} + \Delta\mathbf{v}_{2\min},$$

$$\Delta\mathbf{v}_{1\min} = \sqrt{\left[2U(\mathbf{r}_0) - C_J(\mathbf{r}_0,\mathbf{v}_0)\right] - \left[2U(\mathbf{r}_{L1}) - C_J(\mathbf{r}_{L1},\mathbf{v}_{L1})\right]},$$

$$\Delta\mathbf{v}_{2\min} = \sqrt{\left[2U(\mathbf{r}_0) - C_J(\mathbf{r}_k,\mathbf{v}_k)\right] - \left[2U(\mathbf{r}_{L1}) - C_J(\mathbf{r}_{L1},\mathbf{v}_{L1})\right]}$$



The effective potential in the plane containing the orbit

CR3BP: Linear analysis of dynamics near the libration points

By **linearizing** the CR3BP equations of motion in the **vicinity of collinear libration points**, the following system can be obtained:

$$\begin{pmatrix} \ddot{x}-2\dot{y} \\ \ddot{y}+2\dot{x} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} U_{xx} & U_{xy} & U_{xz} \\ U_{xy} & U_{yy} & U_{yz} \\ U_{xz} & U_{yz} & U_{zz} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \dot{\mathbf{r}} \\ \dot{\mathbf{v}} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \mathbf{r} \\ \mathbf{v} \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2c_2+1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1-c_2 & 0 & -2 & 0 & 0 \\ 0 & 0 & -c_2 & 0 & 0 & 0 \end{pmatrix}.$$

The **eigenvalues** of the matrix **A**:

$$\lambda_{1,2} = \pm s = \pm\sqrt{\frac{c_2 - 2 + \sqrt{c_2(9c_2-8)}}{2}},$$

$$\lambda_{3,4} = \pm\omega_p i = \pm\sqrt{\frac{c_2 - 2 - \sqrt{c_2(9c_2-8)}}{2}},$$

$$\lambda_{5,6} = \pm\omega_v i = \pm\sqrt{c_2},$$

$$c_2 = \begin{cases} \dfrac{1}{\gamma^3}\left(\mu + \dfrac{(1-\mu)\gamma^3}{(1\pm\gamma)^3}\right), & L_1^-, \quad L_2^+, \\ \dfrac{1}{\gamma^3}\left(1-\mu + \dfrac{\mu\gamma^3}{(1+\gamma)^3}\right), & L_3. \end{cases}$$

$$x_{L1} = 1-\mu-\gamma,$$
$$x_{L2} = 1-\mu+\gamma,$$
$$x_{L3} = -\mu-\gamma.$$

C.C. Conley, Low energy transit orbits in the restricted three-body problems, SIAM Journal on App. Math., Vol. 16, No. 4 (1968)

CR3BP: Linear analysis of dynamics near the libration points

By **linearizing** the CR3BP equations of motion in the **vicinity of collinear libration points**, the following system can be obtained:

$$\begin{cases} \ddot{x} - 2\dot{y} - (2c_2 + 1)x = 0, \\ \ddot{y} + 2\dot{x} - (1 - c_2)y = 0, \\ \ddot{z} + c_2 z = 0. \end{cases}$$

Its solution has the form:

$$\begin{cases} x(t) = \alpha e^{st} + \beta e^{-st} + A_x \cos(\omega_p t + \phi_p), \\ y(t) = k_1 \alpha e^{st} - k_1 \beta e^{-st} - k_2 A_x \sin(\omega_p t + \phi_p), \\ z(t) = A_z \cos(\omega_v t + \phi_v). \end{cases}$$

$$\lambda_{1,2} = \pm s = \pm\sqrt{\frac{c_2 - 2 + \sqrt{c_2(9c_2 - 8)}}{2}}, \qquad k_1 = \frac{s^2 - 1 - 2c_2}{2s},$$

$$\lambda_{3,4} = \pm\omega_p i = \pm\sqrt{\frac{c_2 - 2 - \sqrt{c_2(9c_2 - 8)}}{2}}, \qquad k_2 = \frac{\omega_p^{\ 2} + 1 + 2c_2}{2\omega_p},$$

$$\lambda_{5,6} = \pm\omega_v i = \pm\sqrt{c_2}, \qquad \alpha, \beta, A_x, A_z, \phi_p, \phi_v - const.$$

Transit trajectories in the case: $\alpha\beta < 0$

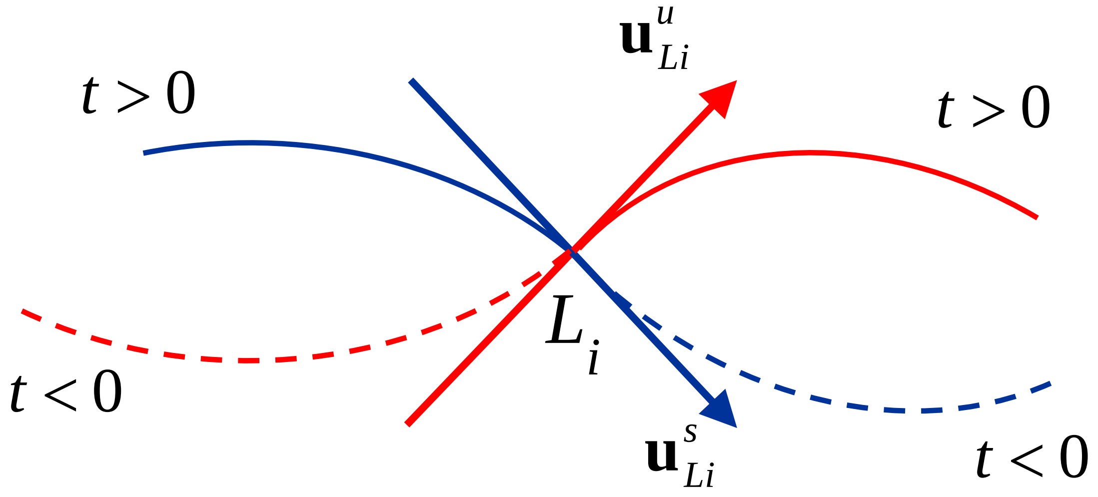
C.C. Conley, Low energy transit orbits in the restricted three-body problems, SIAM Journal on App. Math., Vol. 16, No. 4 (1968)

CR3BP: Linear analysis of dynamics near the libration points

Stable and unstable manifolds:

$$\begin{pmatrix} \mathbf{r}^s_{Li}(0) \\ \mathbf{v}^s_{Li}(0) \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{Li} \\ \mathbf{v}_{Li} \end{pmatrix} \pm \varepsilon \mathbf{u}^s_{Li},$$

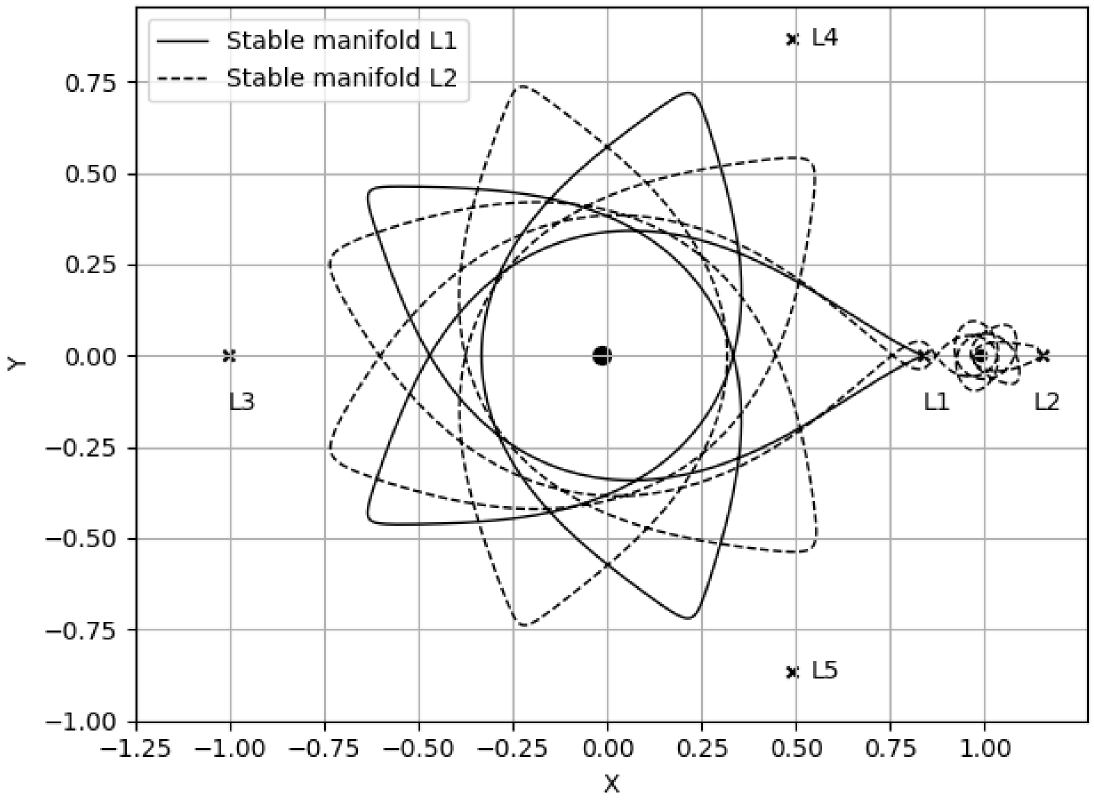
$$\begin{pmatrix} \mathbf{r}^u_{Li}(0) \\ \mathbf{v}^u_{Li}(0) \end{pmatrix} = \begin{pmatrix} \mathbf{r}_{Li} \\ \mathbf{v}_{Li} \end{pmatrix} \pm \varepsilon \mathbf{u}^u_{Li},$$

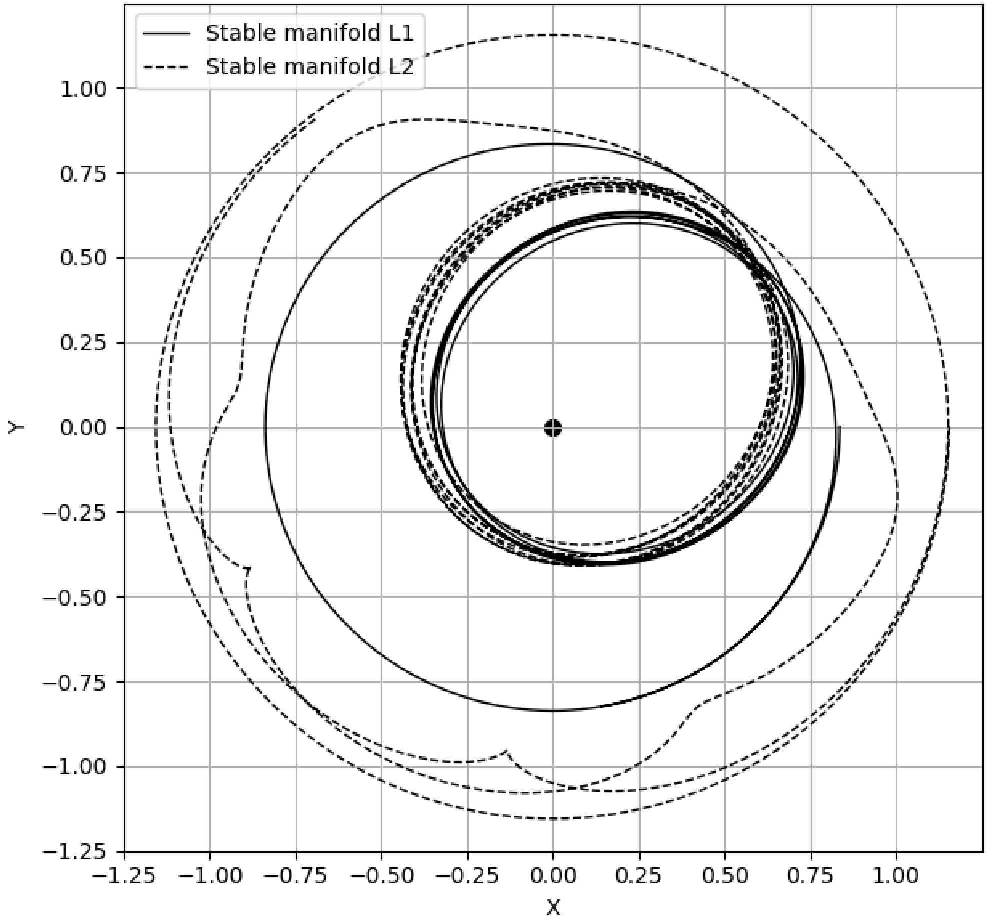


where ε is small (~1.e-6),
$\mathbf{u}^s_{Li}, \mathbf{u}^u_{Li}$ - eigenvectors associated with $\lambda_{1,2} = \pm s$.

The trajectories of a stable manifold tend to Li at $t \to \infty$,
the trajectories of an unstable manifold tend to Li at $t \to -\infty$

CR3BP: Low-energy transit trajectories





Stable manifold L1 (solid line) and L2 (dotted line) in a rotating barycentric coordinate system

(The trajectories of a stable manifold tend to L1 at t $\rightarrow\infty$, the trajectories of an unstable manifold tend to L1 at t $\rightarrow -\infty$)

The elliptic restricted three-body problem (ER3BP)

Equations of motion in a rotating (synodic) system:

$$\begin{cases} \ddot{x} - 2\dot{y} = \dfrac{1}{1+e\cos\upsilon} U_x \\ \ddot{y} + 2\dot{x} = \dfrac{1}{1+e\cos\upsilon} U_y \\ \ddot{z} = \dfrac{1}{1+e\cos\upsilon} U_z \end{cases}$$

where $U = \dfrac{x^2 + y^2}{2} + \dfrac{1-\mu}{r_1} + \dfrac{\mu}{r_2}$,

e, υ – the eccentricity and true anomaly of the orbit of the smaller of the massive bodies.

Obviously, the system has no stationary solutions.

The Jacobi integral is also missing, but a similar integral expression can be written

$$\frac{v^2(t)}{2} - U(t) = \frac{v^2(t_0)}{2} - U(t_0) - \int_{\upsilon(t_0)}^{\upsilon(t)} \frac{\partial U}{\partial \upsilon} d\upsilon.$$

Considering a sufficiently small time interval, the current value of the Jacobi constant can be written using the Jacobi constant from CR3BP

$$C_J^* = C_J (1 + e\cos\upsilon).$$

Szebehely V. Theory of orbits, Academic press, New York and London, 1967

ER3BP: linear analysis of dynamics near the libration points

Transformation to rotating–pulsating coordinates:

$$\begin{cases} x'' - 2y' = \tilde{U}_x \\ y'' + 2x' = \tilde{U}_y \\ z'' = \tilde{U}_z \end{cases}$$

where $\tilde{U} = \dfrac{1}{1+e\cos\upsilon}\left(\dfrac{x^2+y^2-ez^2\cos\upsilon}{2} + \dfrac{1-\mu}{r_1} + \dfrac{\mu}{r_2}\right)$,

e, υ – the eccentricity and true anomaly of the orbit of the smaller of the massive bodies.

By linearizing the ER3BP equations of motion in the vicinity of collinear libration points, the following system can be obtained:

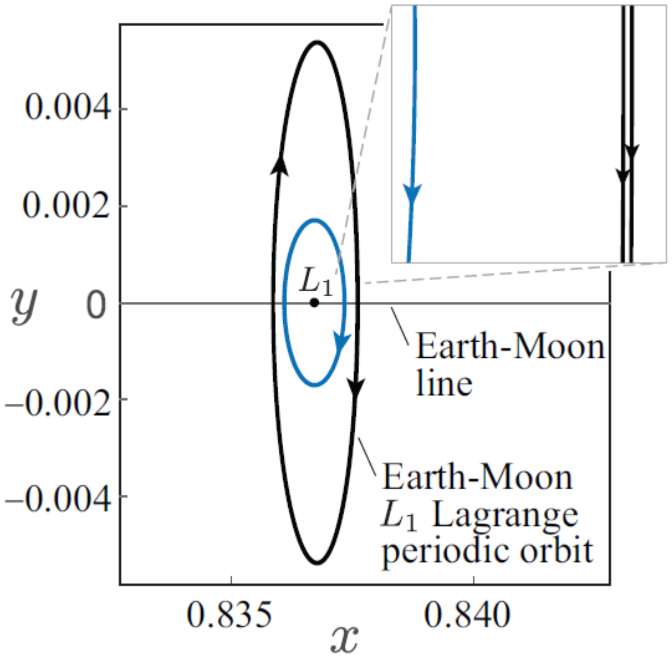
$$\begin{pmatrix} \ddot{x} - 2\dot{y} \\ \ddot{y} + 2\dot{x} \\ \ddot{z} \end{pmatrix} = \frac{1}{1+e\cos\upsilon}\begin{pmatrix} 2c_2+1 & 0 & 0 \\ 0 & 1-c_2 & 0 \\ 0 & 0 & -c_2 - e\cos\upsilon \end{pmatrix}.$$

In this case, the eigenvalues are not constant and are periodic functions of the true anomaly:

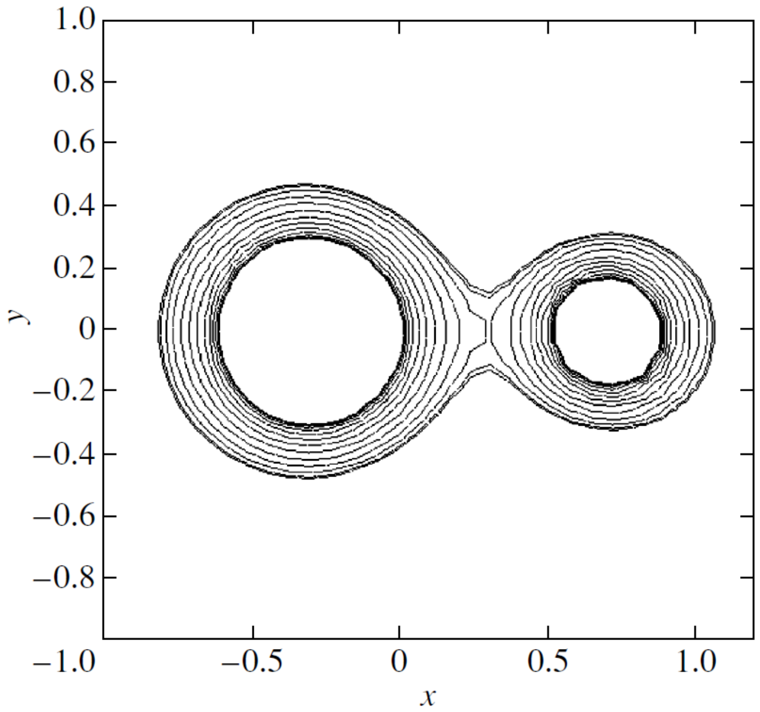
$$\lambda_{1,2} = \pm s(\upsilon), \quad \lambda_{3,4} = \pm\omega_p(\upsilon)i, \quad \lambda_{5,6} = \pm\omega_v(\upsilon)i.$$

Szebehely V. Theory of orbits, Academic press, New York and London, 1967;
Farquhar R.W. The control and use of libration-point satellites, 1970.

ER3BP: Libration points and Jacobi integral



Fitzgerald J., Ross S. D. Geometry of transit orbits in the periodically-perturbed restricted three-body problem // Advances in Space Research, 2022, Vol. 70, № 1, pp. 144-156.



Luk'Yanov L.G. Energy conservation in the restricted elliptical three-body problem // Astronomy reports, 2005, Vol. 49, pp. 1018-1027

ER3BP: linear dynamics near the libration points

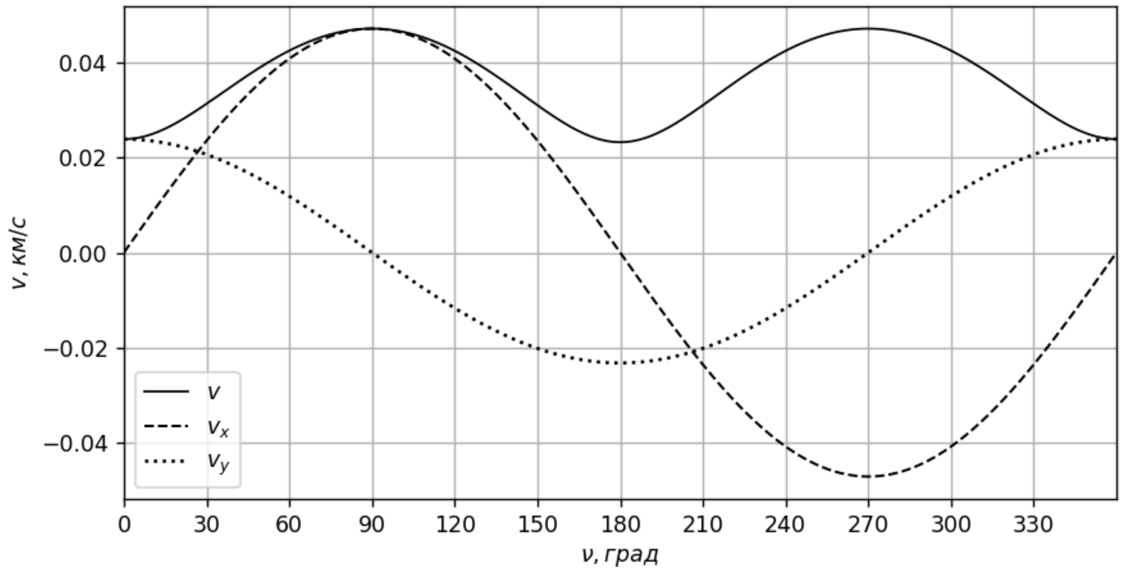
Since the rotating coordinate system has a local circular velocity, taking into account the orientation of the axes, the velocity of the libration point in the rotating coordinate system can be determined as follows:

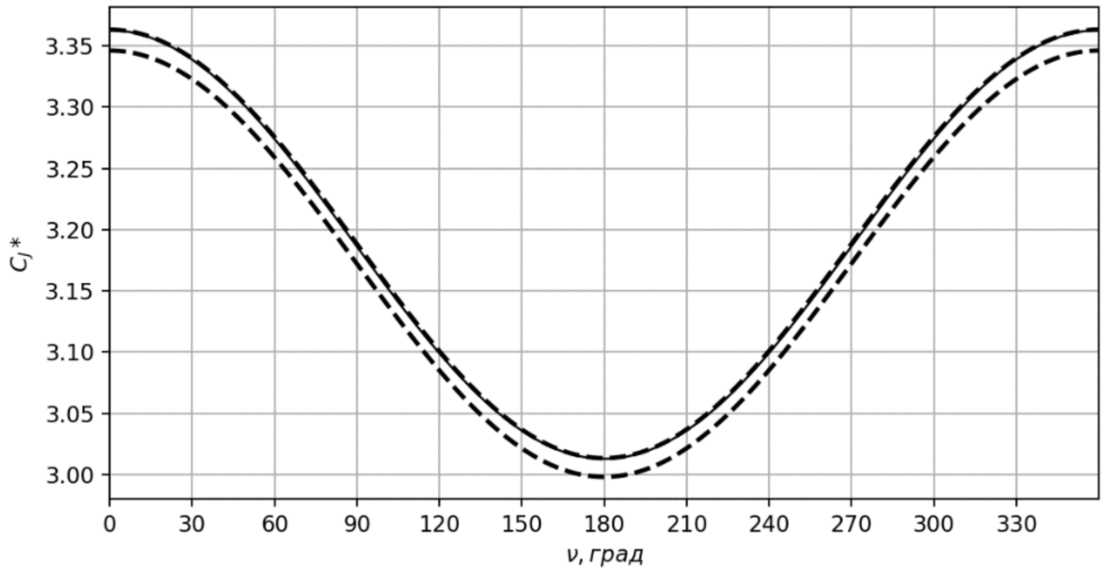
$$v_x^{Li} = x_{Li}\sqrt{\frac{\mu_E + \mu_M}{p}} e \sin\upsilon,$$

$$v_y^{Li} = x_{Li}\sqrt{\frac{\mu_E + \mu_M}{p}} \left(1 + e\cos\upsilon - \sqrt{1 + e\cos\upsilon}\right),$$

$$v_z^{Li} = 0.$$

where e, p, υ – eccentricity, semi-latus rectum and true anomaly of the Moon's orbit.

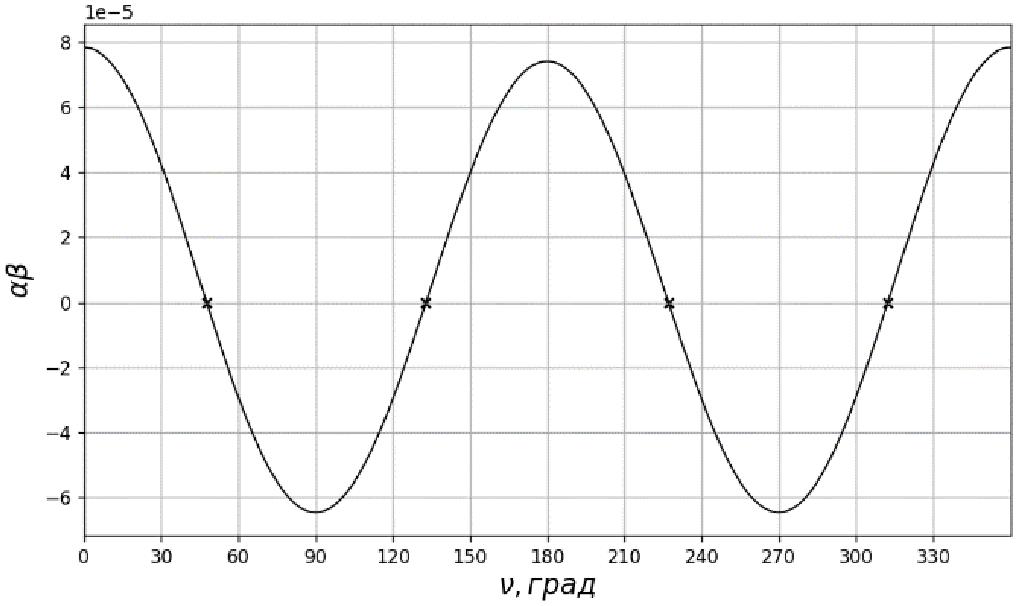


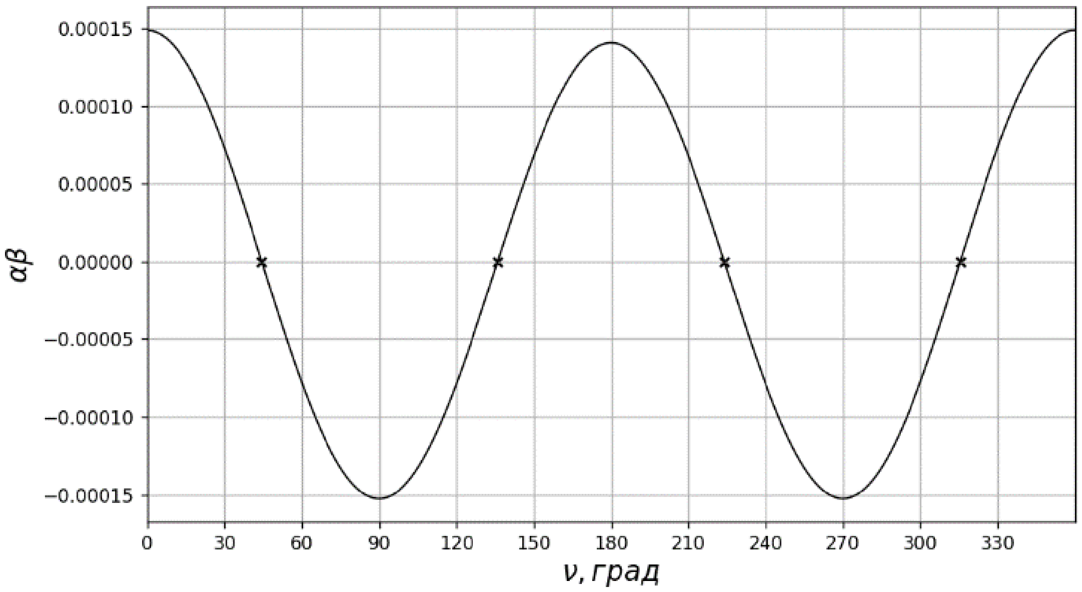


ER3BP: linear dynamics near the libration points

In the instantaneous CR3BP, libration points have a velocity by which the type of motion can be determined within the framework of a linear model from the following equations:

$$\begin{pmatrix} s & -s \\ k_1 s & k_1 s \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}, \qquad \text{where} \quad \begin{pmatrix} V_x \\ V_y \end{pmatrix} = x_L \begin{pmatrix} V_r \\ V_n - V_{кр.} \end{pmatrix}.$$

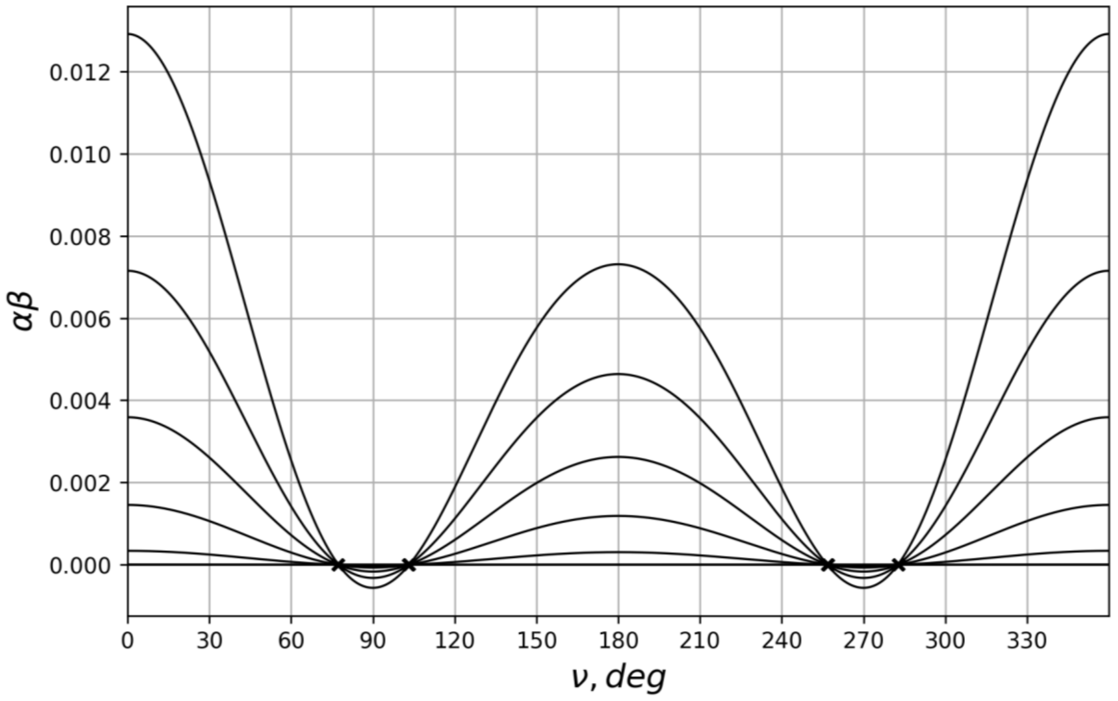


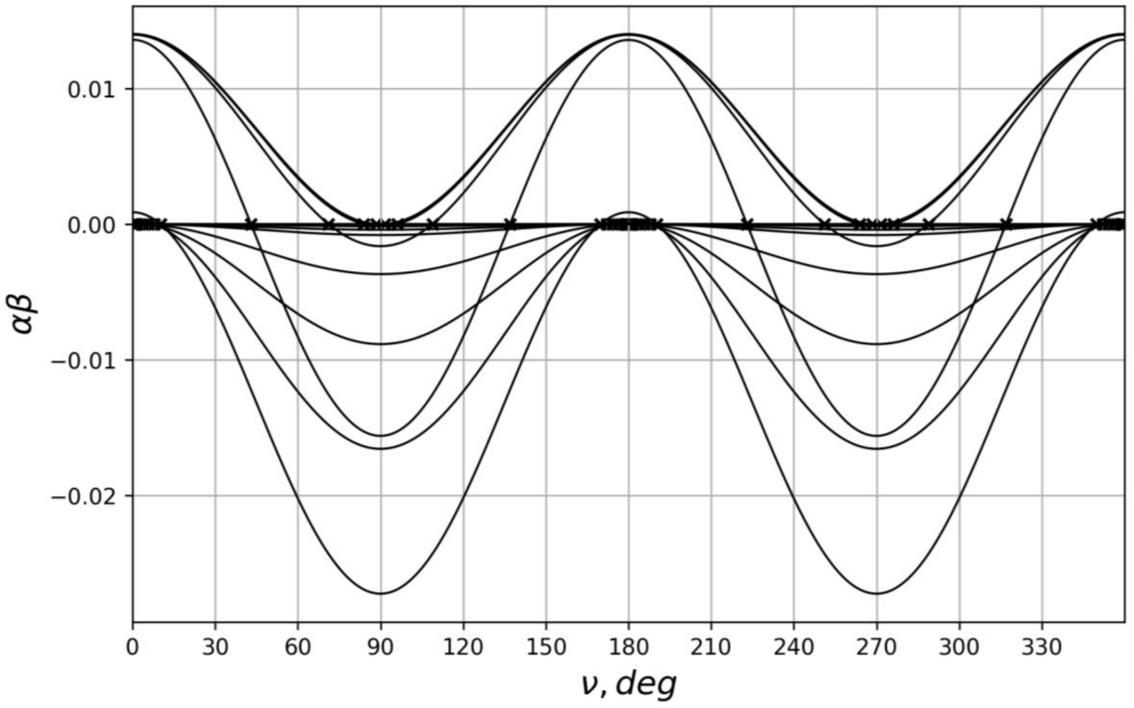


The true anomaly of the Moon's orbit:

- to Moon (L1) from - 47° to - 132°; (L2) from 44° to 136°.
- from Moon (L1) from 47° to 132°; (L2) from - 44° to - 136°.

ER3BP: linear dynamics near the libration points





μ = 0.001
e = 0.001, 0.01, 0.1, 0.2, 0.3, 0.4, 0.5
77° – 103° and 257° – 283°

μ = 1.e-06 43° - 137° and 223° - 317°
μ = 1.e-05 71° - 109° and 251° - 289°
μ = 0.0001 84° - 96° and 264° – 276°
μ = 0.001 88° - 92° and 268° - 272°
μ = 0.01 0° - 180° and 180° - 360°
μ = 0.1 2° - 178° and 182° - 358°

The restricted four-body problem (R4BP)

$$\ddot{\mathbf{r}} = -\frac{\mu_E \mathbf{r}}{|\mathbf{r}|^3} + \mu_S \left(\frac{\mathbf{r}-\mathbf{r}_S}{|\mathbf{r}-\mathbf{r}_S|} + \frac{\mathbf{r}_S}{|\mathbf{r}_S|} \right) + \mu_M \left(\frac{\mathbf{r}-\mathbf{r}_M}{|\mathbf{r}-\mathbf{r}_M|} + \frac{\mathbf{r}_M}{|\mathbf{r}_M|} \right),$$

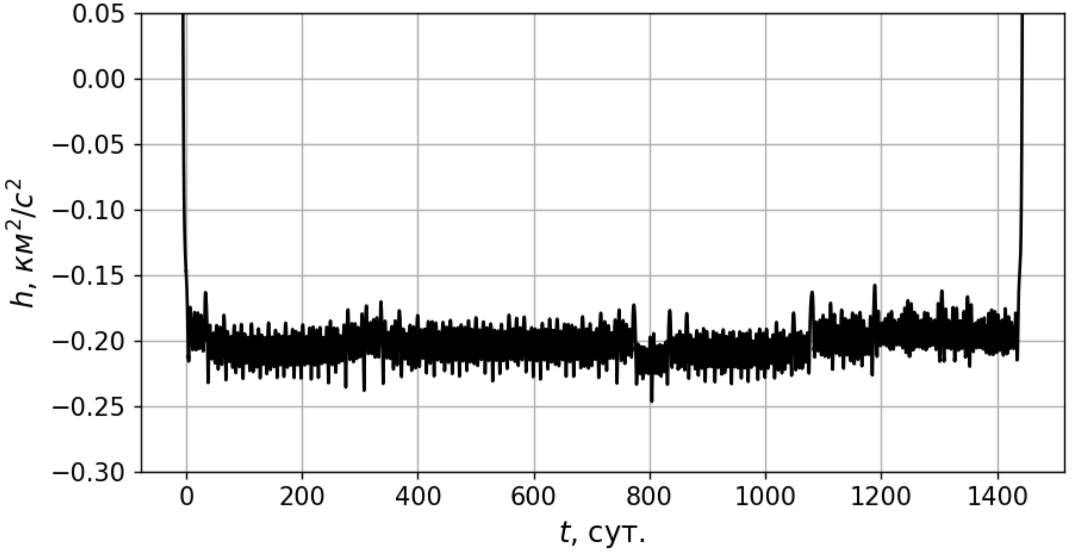
where μ_E, μ_S, μ_M - gravitational parameters of the Earth, the Sun and the Moon, $\mathbf{r}_S$, $\mathbf{r}_M$ – geocentric position vectors of the Sun and the Moon, respectively. The position and velocity vectors of the Sun and the Moon are calculated using ephemeris software JPL DE405.

The instantaneous synodic coordinate system related to the current position and velocity of the Moon in the inertial geocentric coordinate system J2000 as follows:

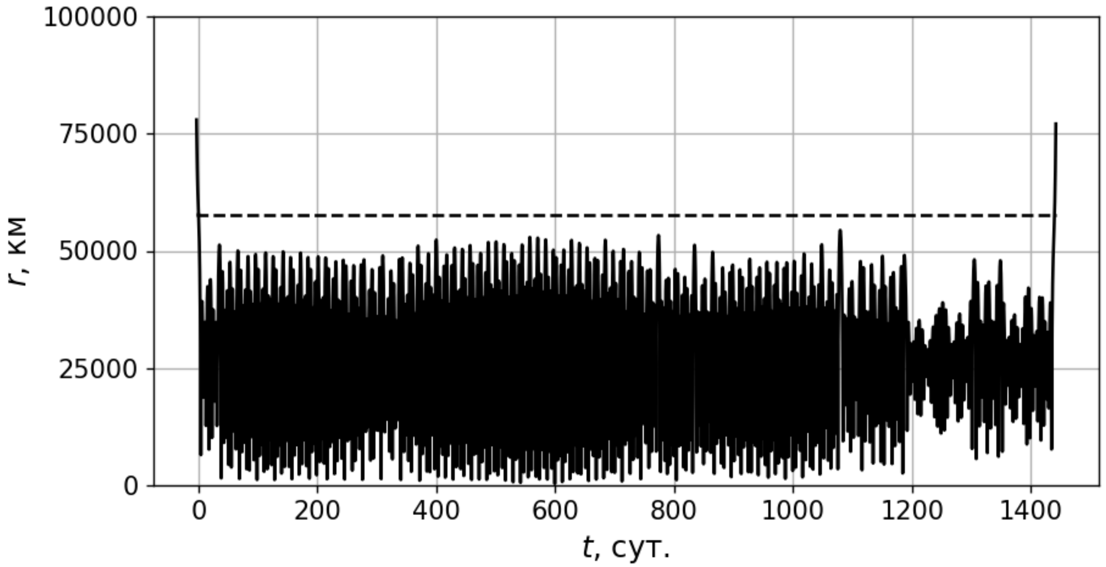
$$\begin{pmatrix} \mathbf{r}_{J2000}(t) \\ \mathbf{v}_{J2000}(t) \end{pmatrix} = \mathbf{M}(t) \begin{pmatrix} \mathbf{r}(t) \\ \mathbf{v}(t) \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z & 0 & 0 & 0 \\ \dot{\vartheta}\mathbf{e}_y & -\dot{\vartheta}\mathbf{e}_x & 0 & \mathbf{e}_x & \mathbf{e}_x & \mathbf{e}_x \end{pmatrix} \begin{pmatrix} |\mathbf{r}_M| & 0 \\ 0 & |\mathbf{v}_M| \end{pmatrix},$$

$$\mathbf{e}_x = \frac{\mathbf{r}_M}{|\mathbf{r}_M|}, \quad \mathbf{e}_z = \frac{\mathbf{r}_M \times \mathbf{v}_M}{|\mathbf{r}_M \times \mathbf{v}_M|}, \quad \mathbf{e}_y = \mathbf{e}_z \times \mathbf{e}_x, \quad \dot{\vartheta} = \frac{|\mathbf{r}_M \times \mathbf{v}_M|}{|\mathbf{r}_M|^2}.$$

Case Earth – Moon 1: Transit trajectory of long-term capture



Dependence of selenocentric Keplerian energy on the trajectory of temporary capture.

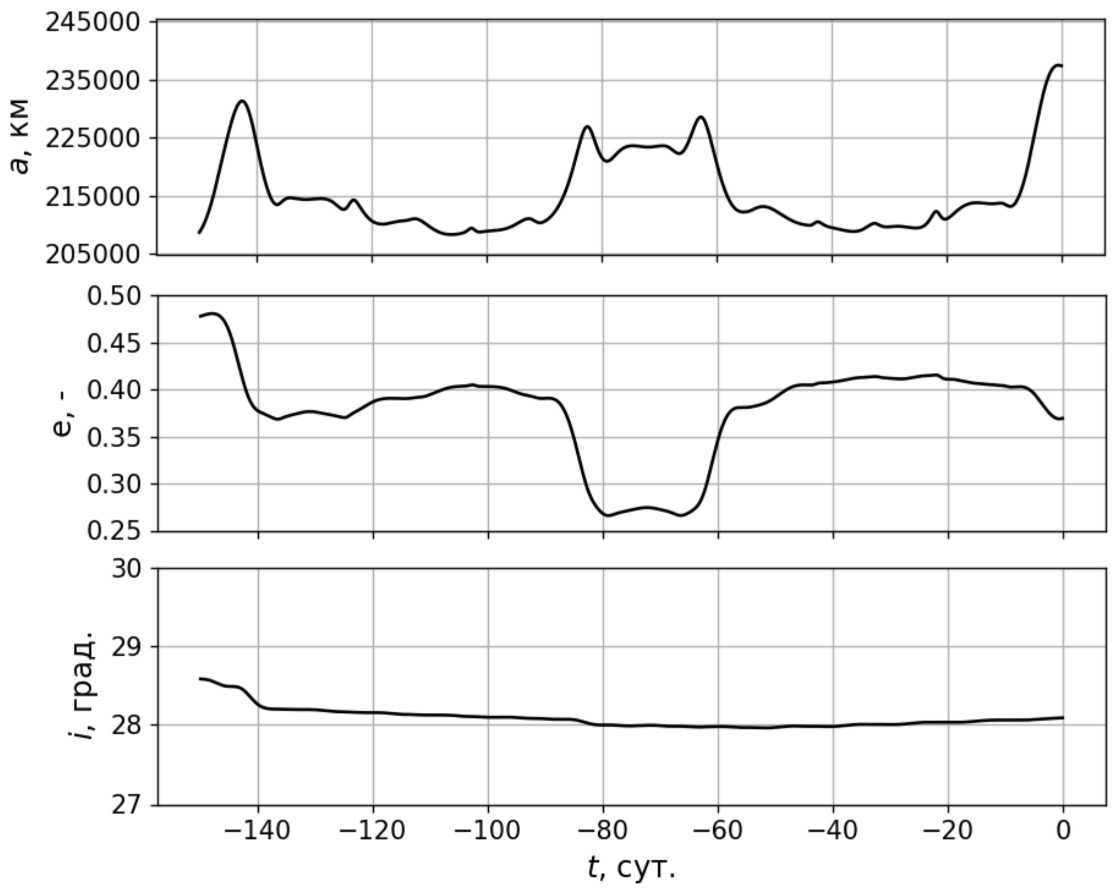


Dependence of the distance to the Moon on the trajectory of temporary capture.

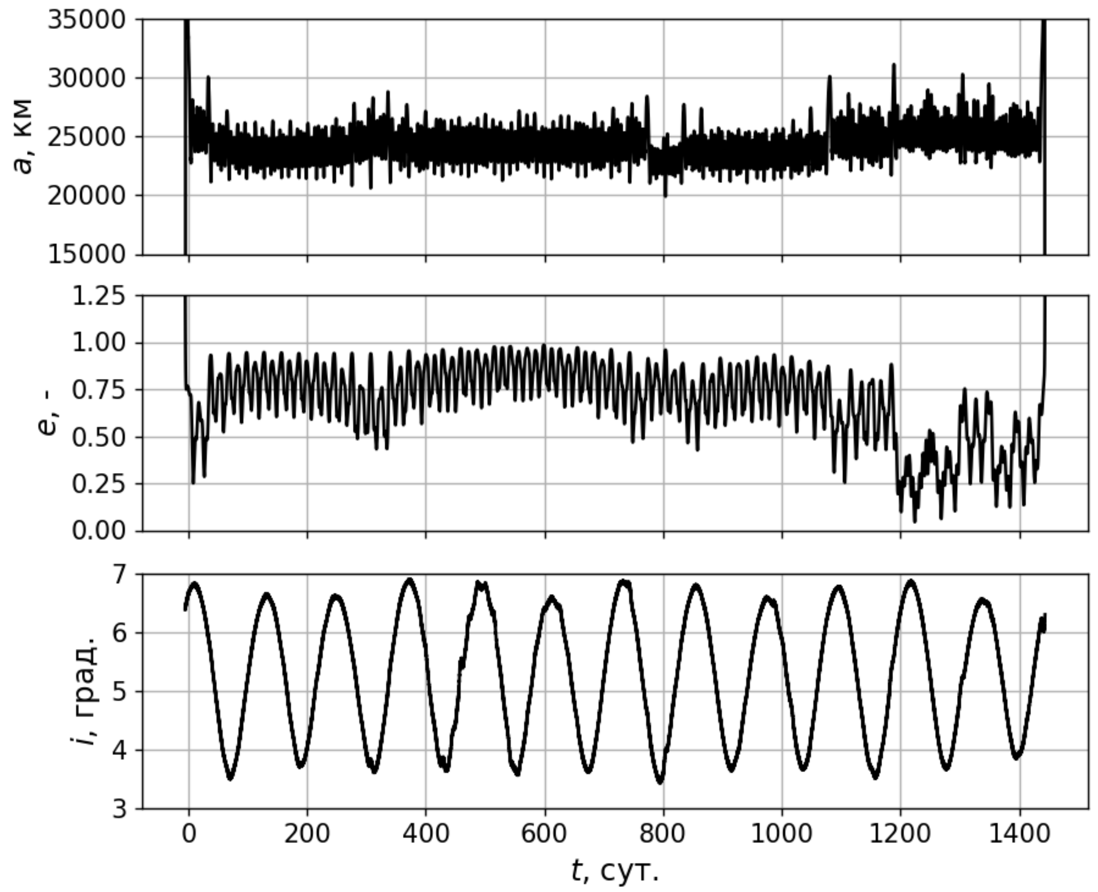
Formed by a stable and unstable **L1** manifold of the Earth – Moon system with a moment of passing **L1** 03.08.2026 15:05:50 (JD 2461256.12905)

Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Designing Low-energy low-thrust flight to the Moon on a temporary capture trajectory // Cosmic Research, Vol. 61, No. 5 (2023) 380-393

Case Earth – Moon 1: Transit trajectory of long-term capture



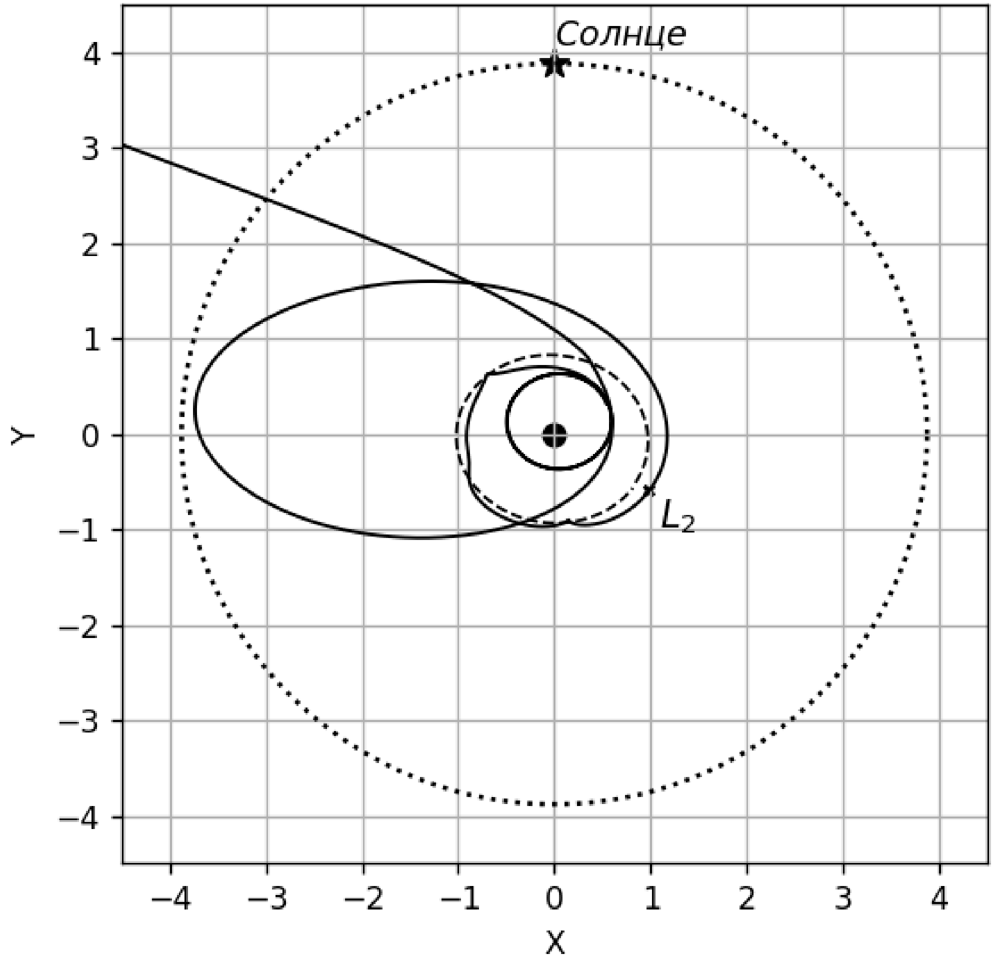
relative to the Earth.

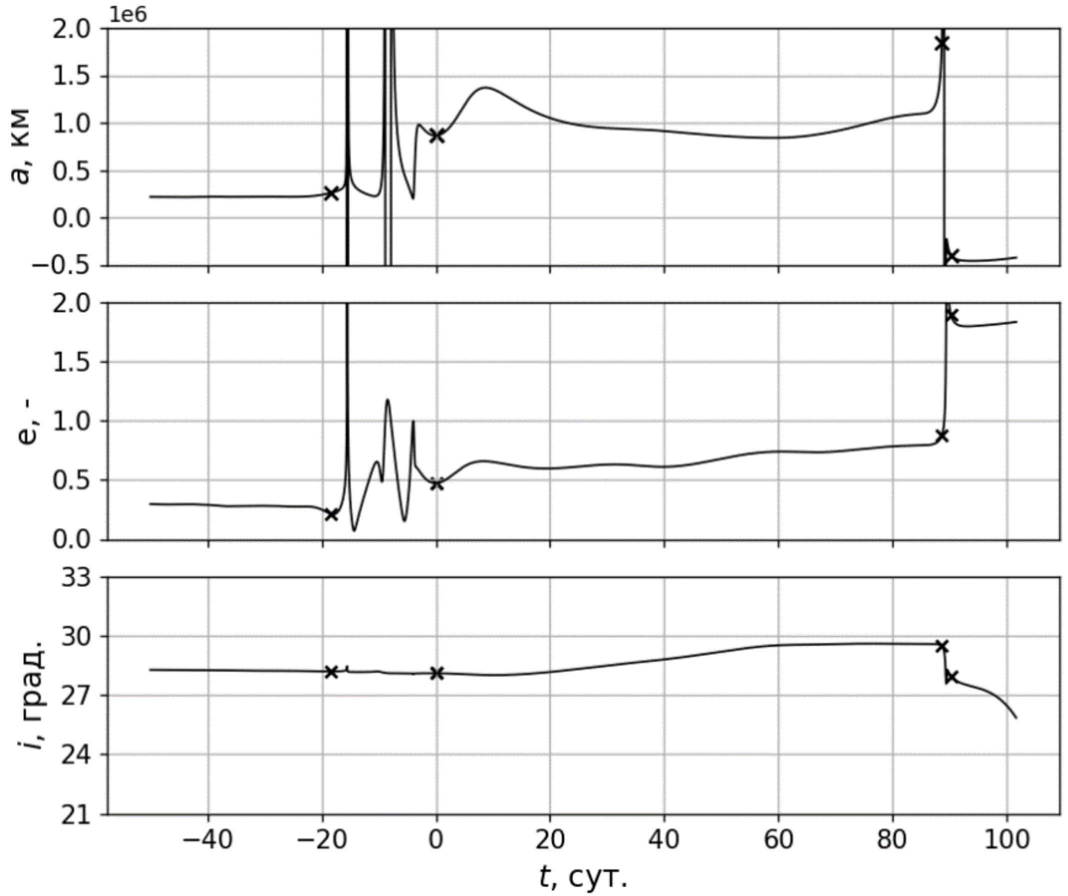


relative to the Moon.

Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Designing Low-energy low-thrust flight to the Moon on a temporary capture trajectory // Cosmic Research, Vol. 61, No. 5 (2023) 380-393

Case Earth – Moon 2: Departure transit trajectory of temporary capture



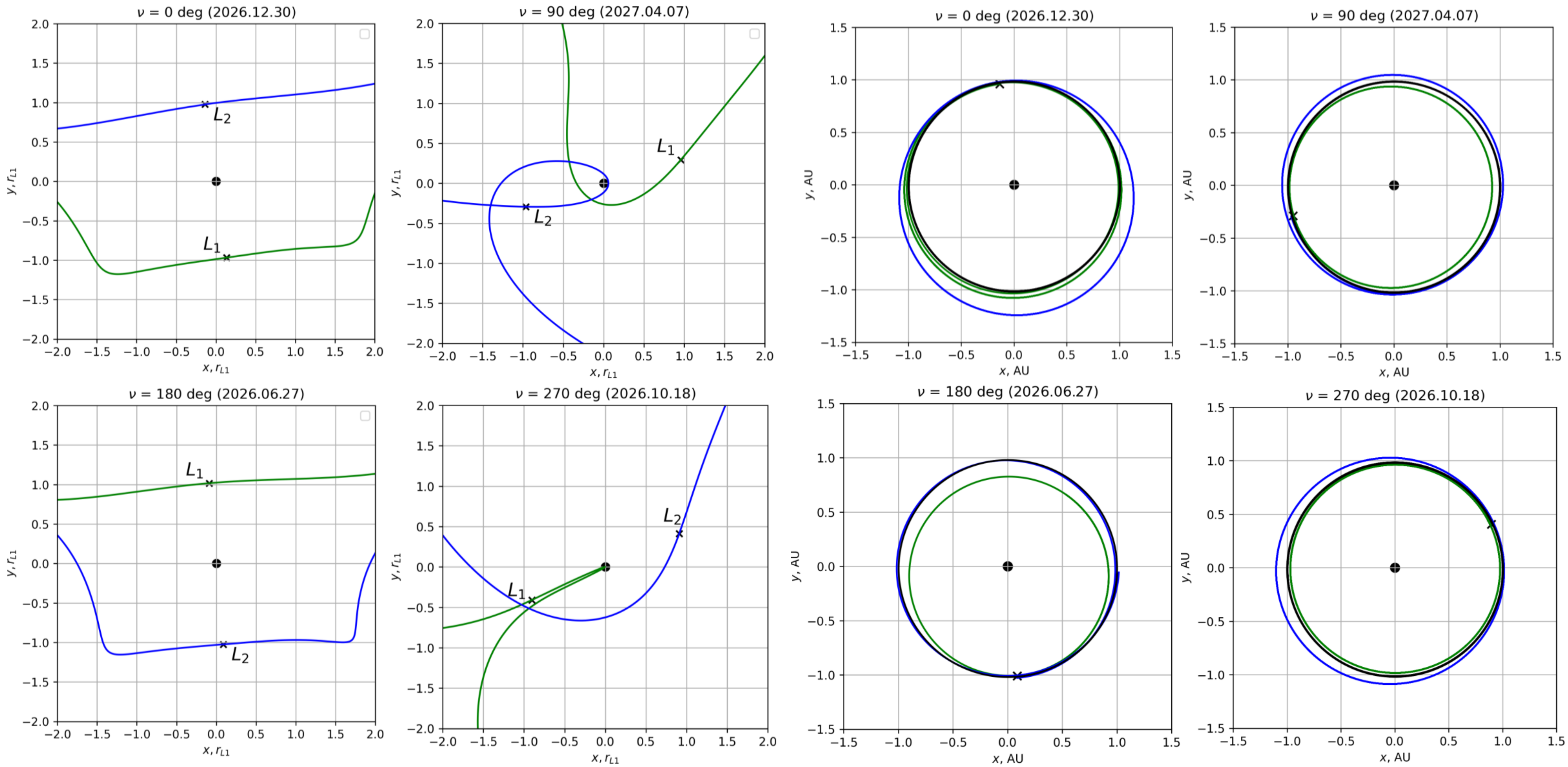


relative to the Earth.

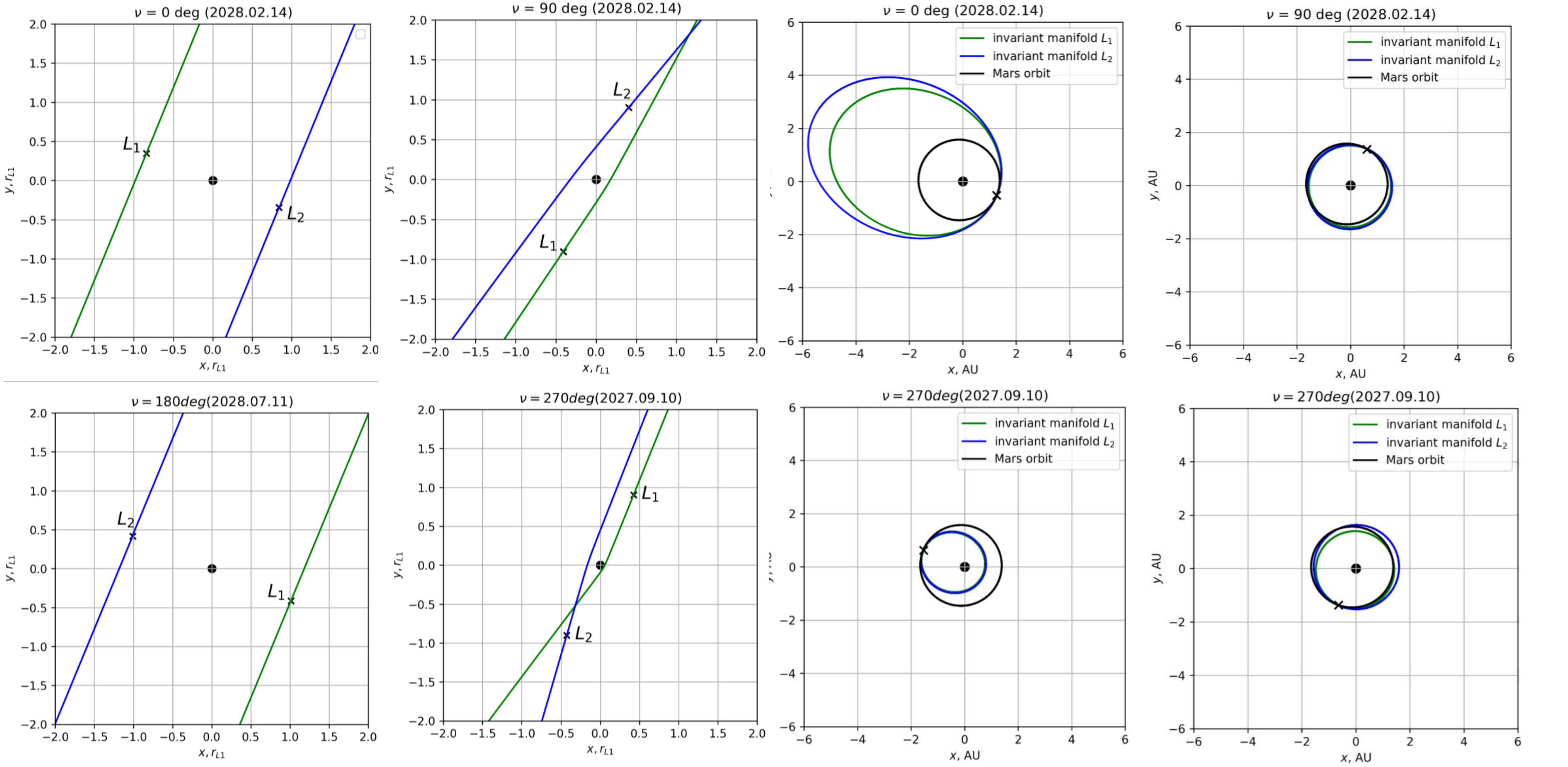
Formed by a stable and unstable **L2** manifold of the Earth – Moon system with a moment of passing **L2** **10.05.2026 11:34:16**. The velocity of spacecraft when crossing the SOI of the Earth: **1.3 km/s.**

Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Designing Low-energy low-thrust flight to the Moon on a temporary capture trajectory // Cosmic Research, Vol. 61, No. 5 (2023) 380-393

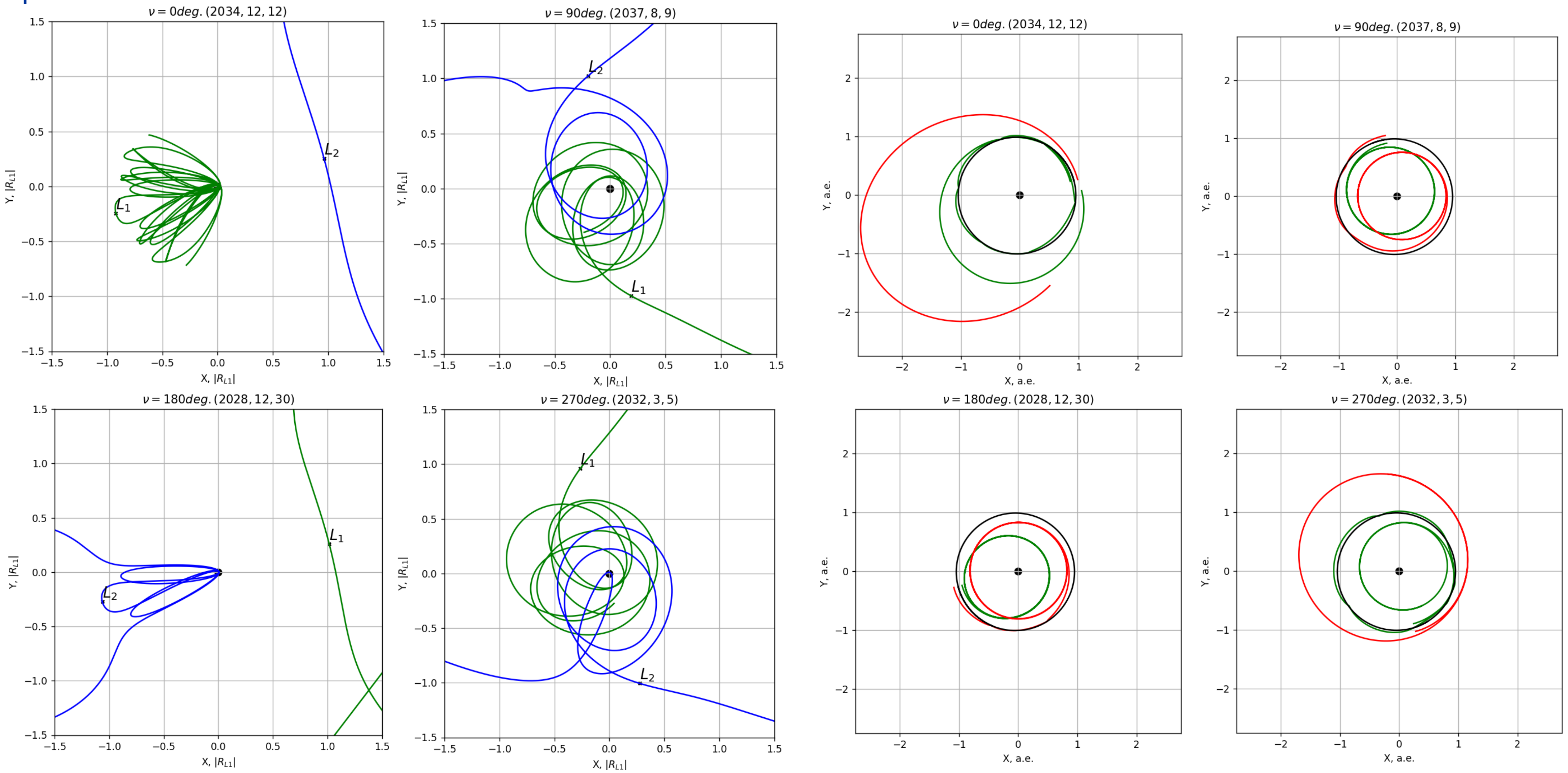
Case Sun-Earth: Transit trajectories



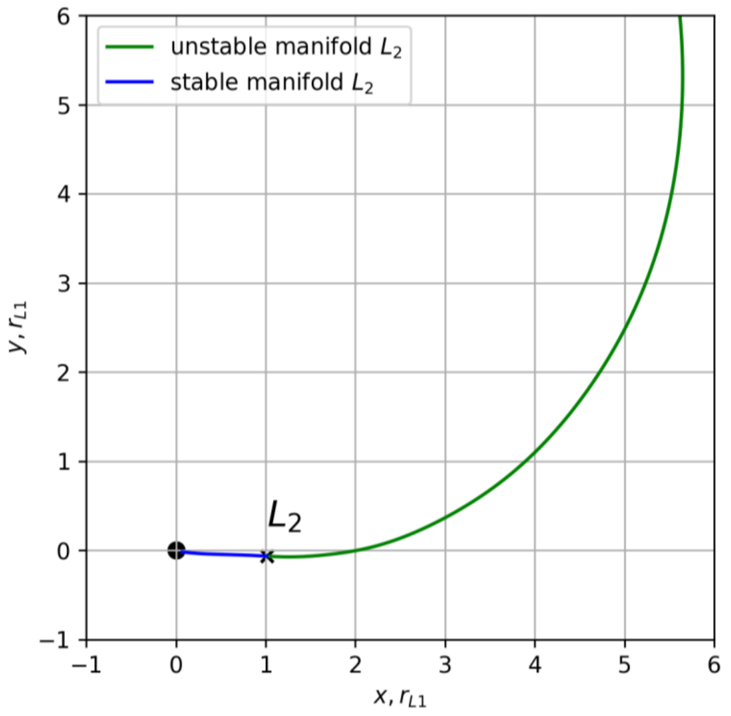
Case Sun-Mars: Transit trajectories

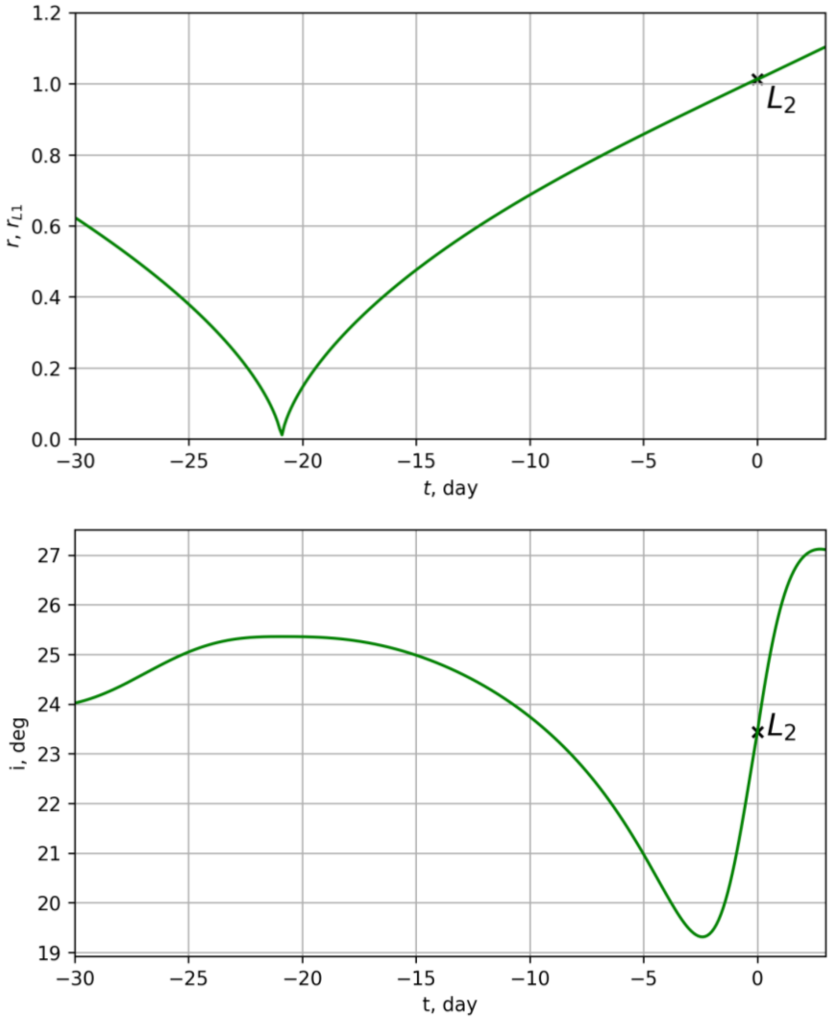


Case Sun-Jupiter: Transit trajectories



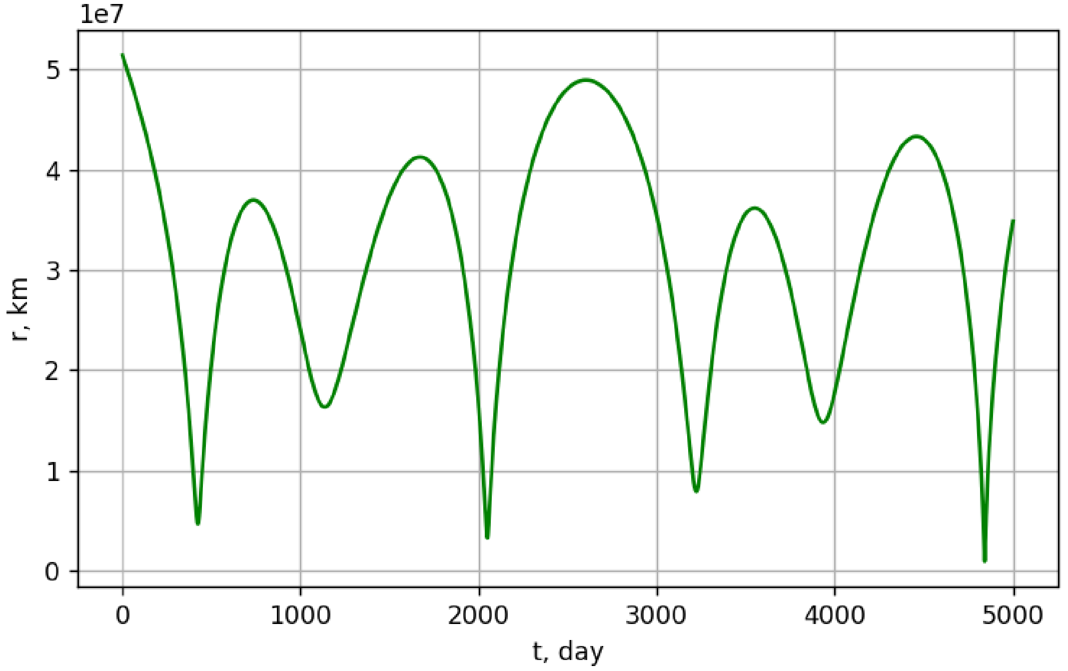
Case Sun-Earth: Departure transit trajectory from Earth

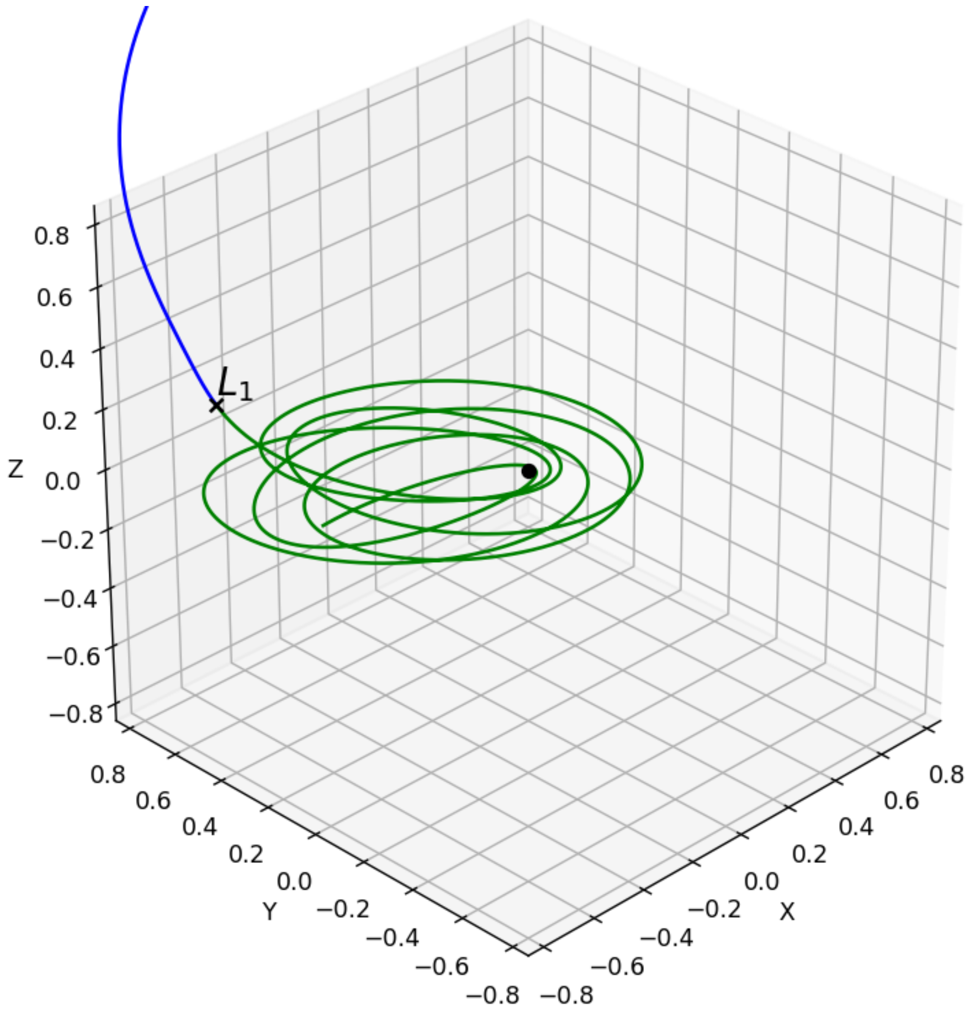




min distance	**16 495**	**km**
passage date of libration point	**2026.09.17**	

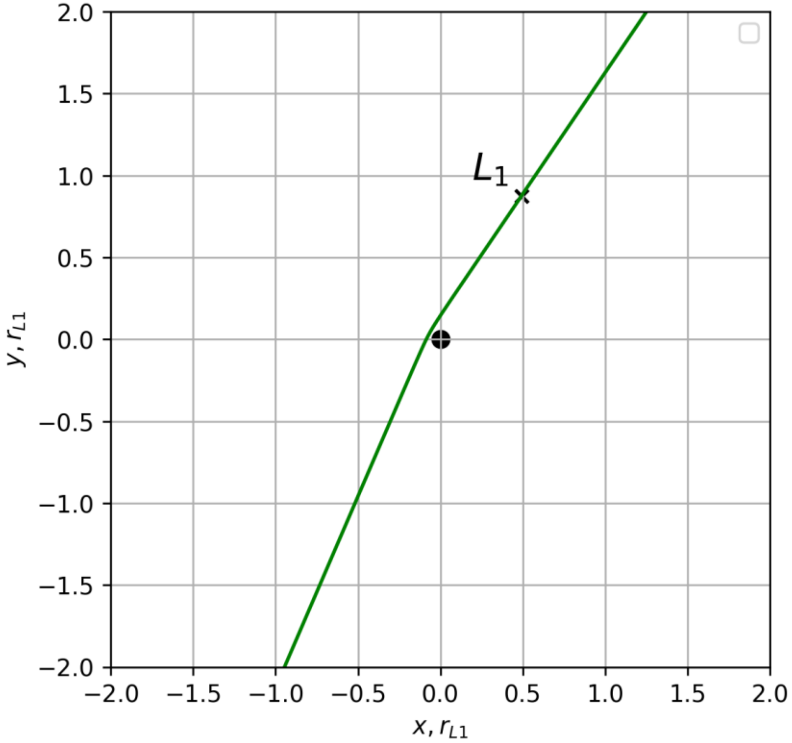
Case Sun-Jupiter: Approach transit trajectory

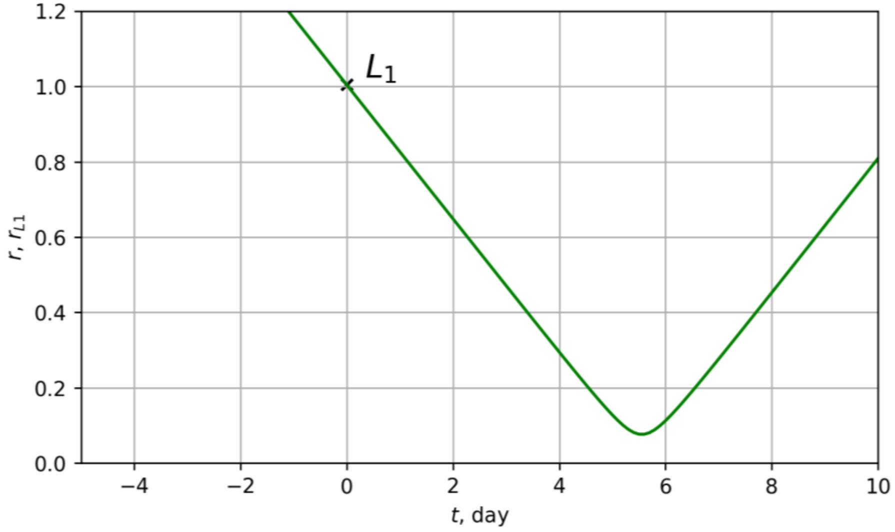


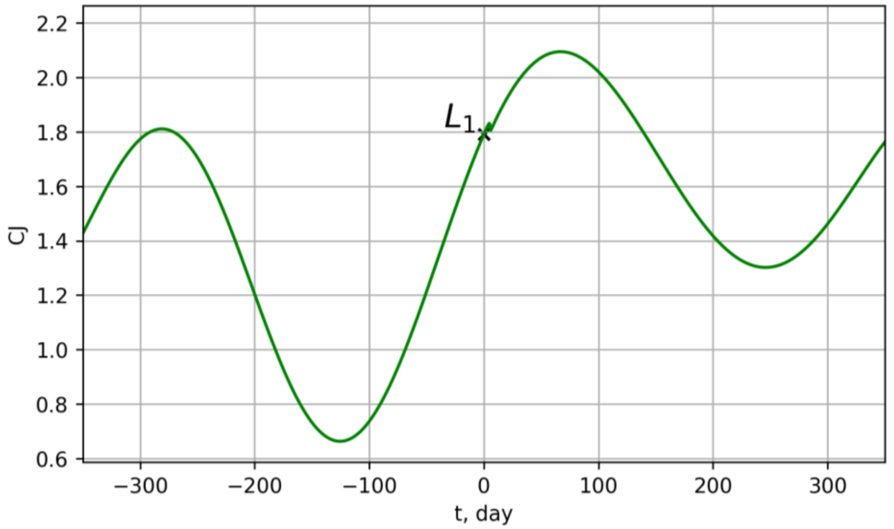


min distance **954 632** **km**
passage date of libration point **2026.09.17**

Case Sun-Mars: Approach transit trajectory

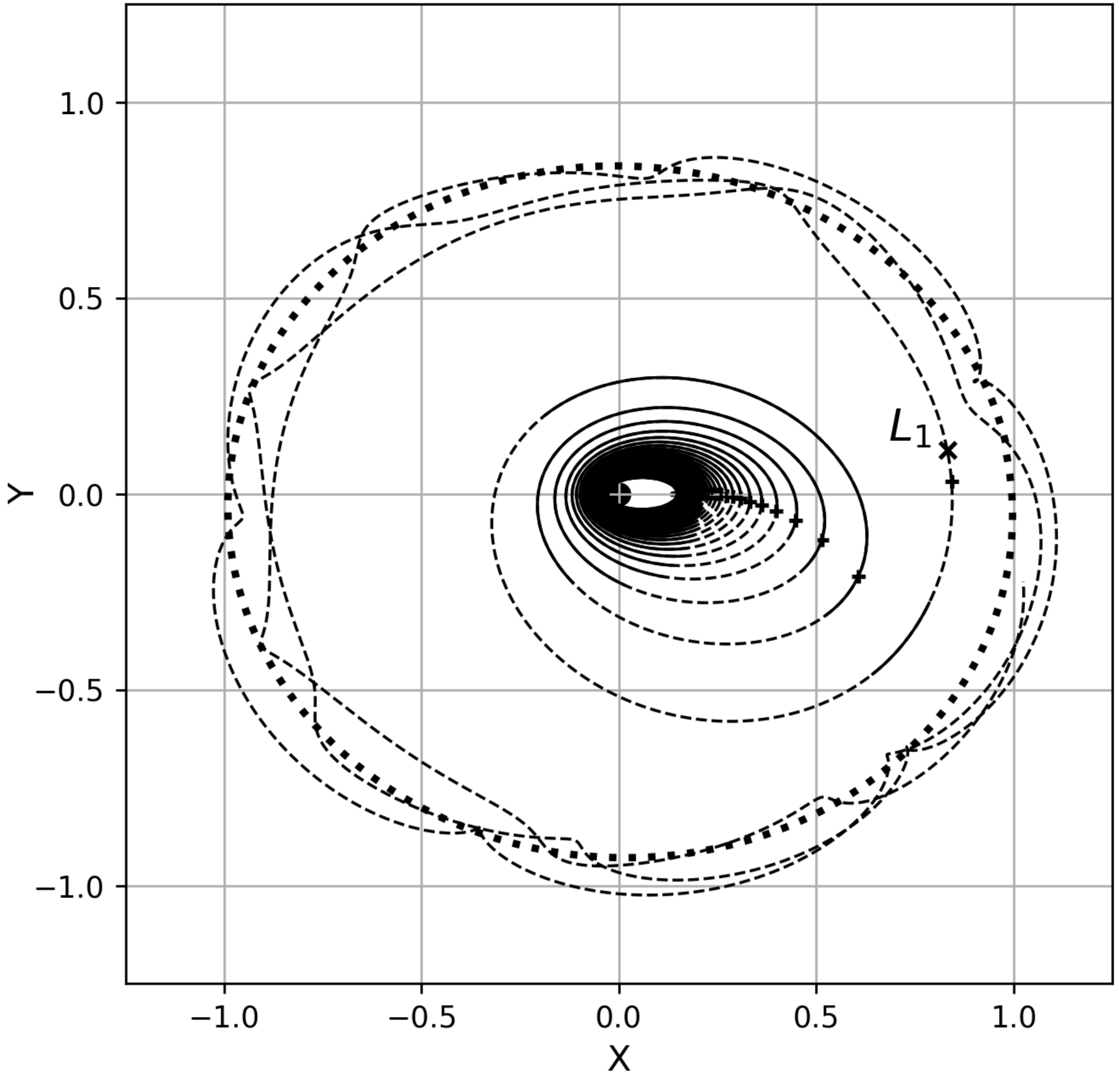


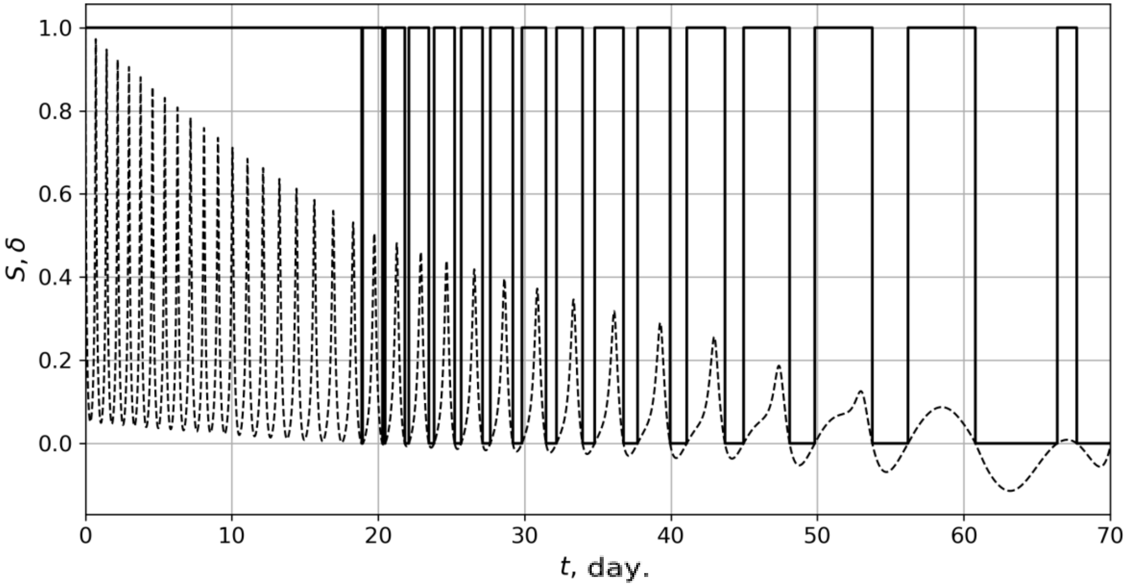




min distance	**82 488**	**km**
passage date of libration point	**2027.09.02**	

Case Earth-Moon 1: GTO and reduction of transfer duration





Initial orbit:

r_0	6 621	km
$i\ /\ \Omega\ /\ \omega$	51.6 / 0 / 0	deg

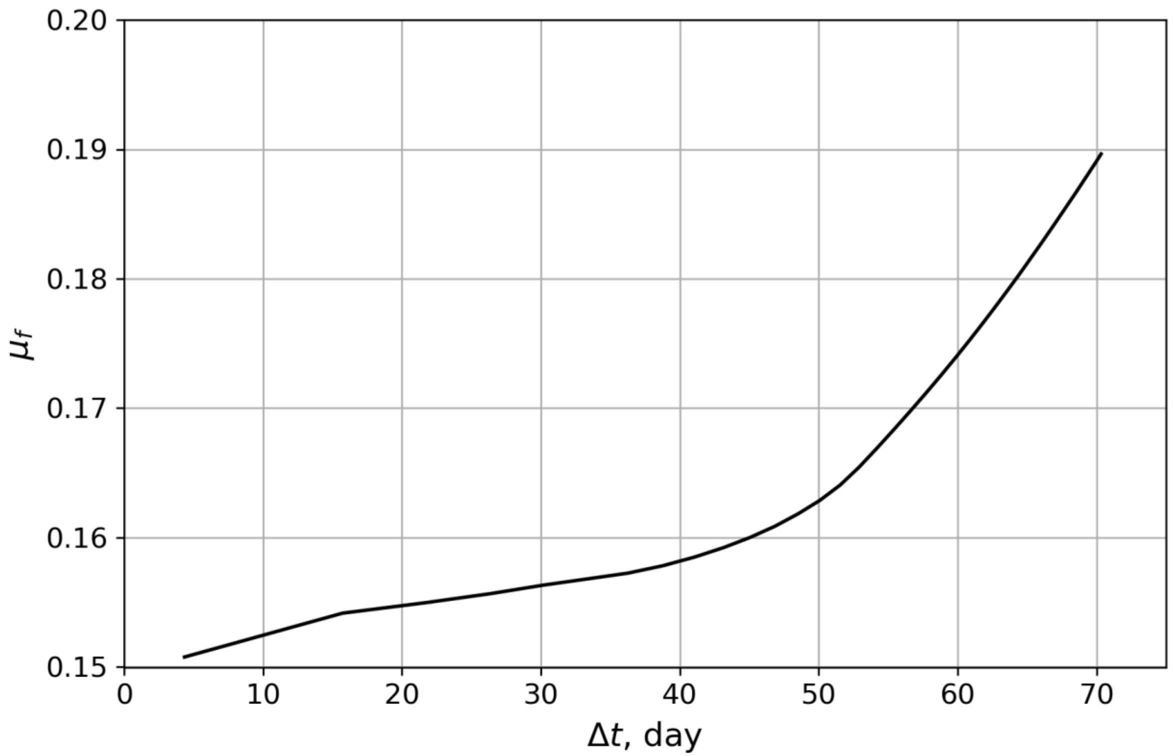
Intermediate orbit:

$r_\pi\ /\ r_\alpha$	10 871 / 56 371	km
$i\ /\ \Omega\ /\ \omega$	51.6 / 0 / 180	deg

Angular distance of 30 revolutions. The transfer duration is 70 days.

Case Earth-Moon 1: GTO and reduction of transfer duration

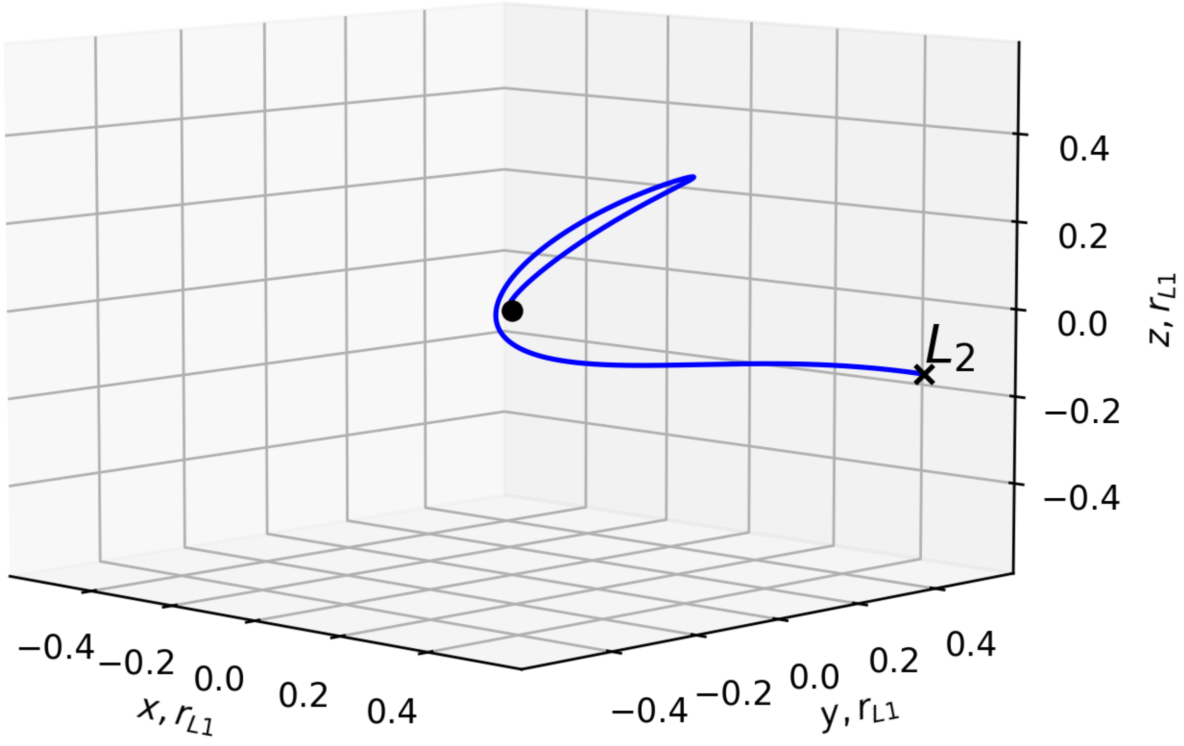
To transfer to the Moon on the trajectory of long-term capture, a combined flight scheme was analyzed



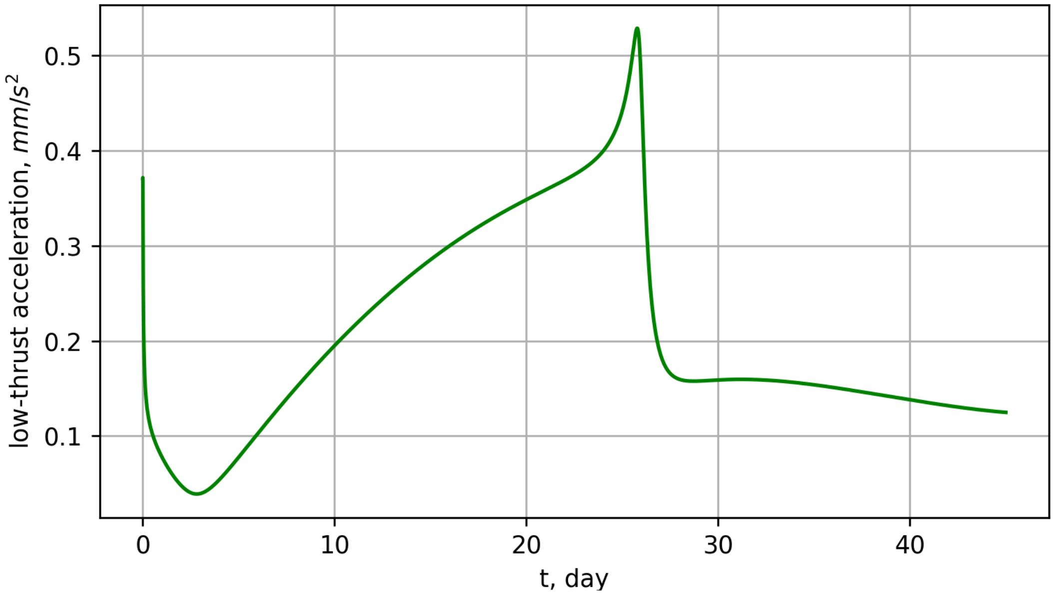
Δt, days	r_α, km	V_α, km/s	Δi, deg.	ΔV_Σ, km/s	m_{0}, kg	μ_f
70.3	56371	1.512	0.0	2.914	4882	0.190
65.3	61875	1.510	1.4	3.065	5094	0.181
59.8	69253	1.495	3.1	3.222	5322	0.173
55.4	76482	1.474	4.6	3.338	5497	0.168
50.0	87188	1.433	6.3	3.460	5683	0.162
45.0	99575	1.355	7.4	3.528	5785	0.160
41.0	111305	1.286	8.3	3.566	5840	0.158
36.2	127966	1.199	9.4	3.600	5887	0.157
30.1	154682	1.088	11.1	3.630	5921	0.156
26.4	174678	1.031	12.7	3.650	5946	0.155
21.7	203853	0.966	15.0	3.673	5973	0.155
15.7	249920	0.895	18.4	3.701	6005	0.154
4.3	325067	0.880	23.5	3.787	6140	0.150

Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Low-Energy Lunar Transfer Design Using High- And Low-Thrust on Ballistic Capture Trajectories // International Astronautical Congress 2023, IAC-23.C1.9.7

Case Sun-Jupiter: Flight near the Earth (LEO – Earth L2)



Trajectory LEO - Earth L2



Low-thrust acceleration

45 day (LEO – Earth L2)

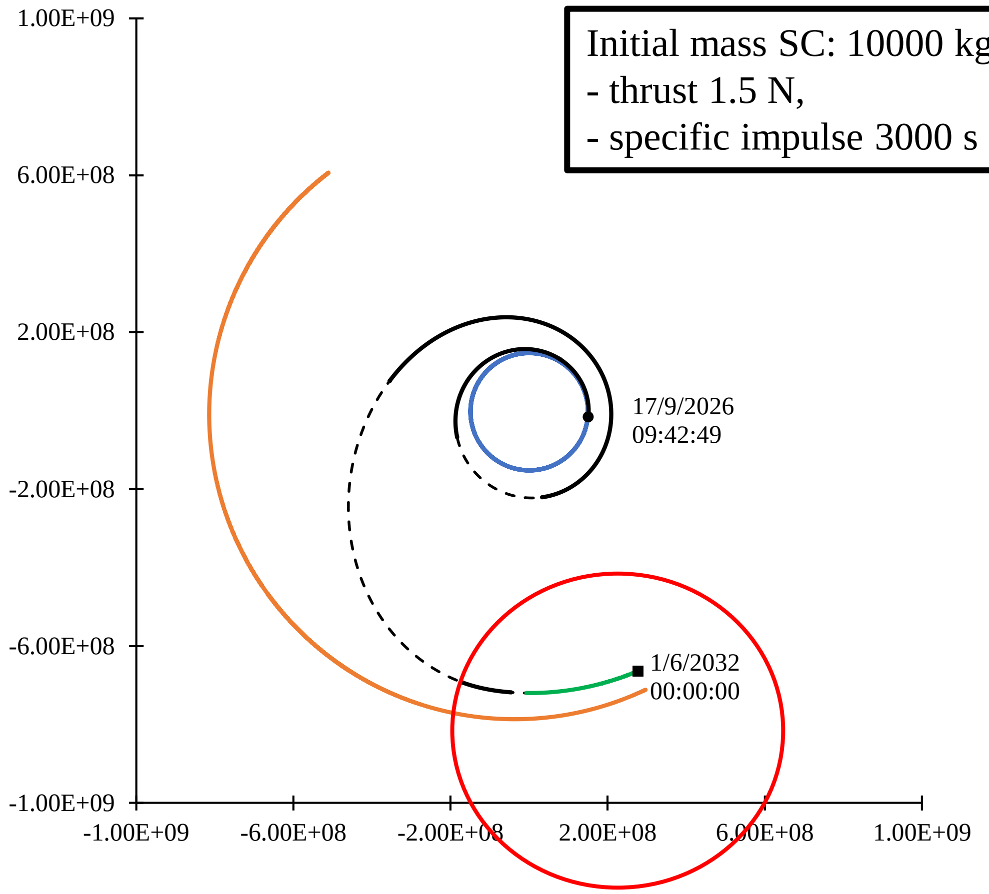
J_{LP} : 0.102575 m^2/s^3

ΔV : 790 m/s

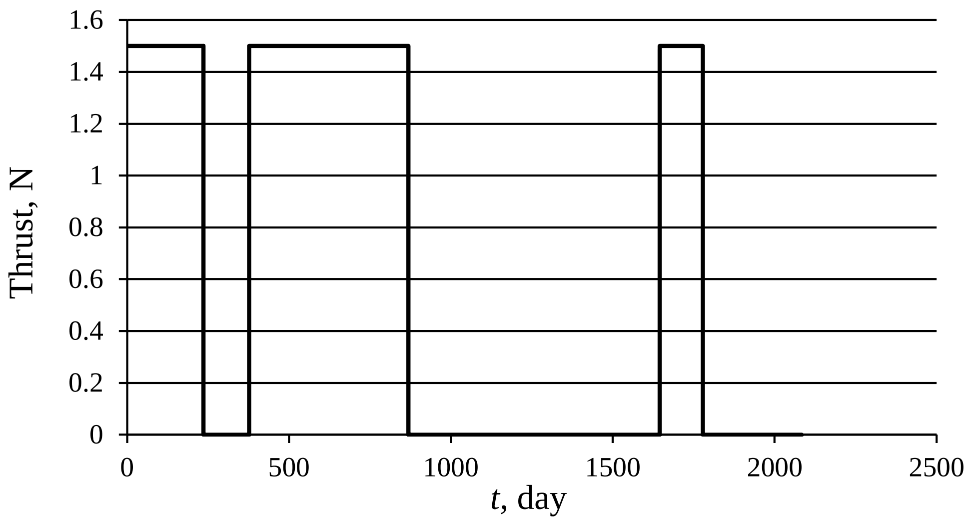
LEO parameters:

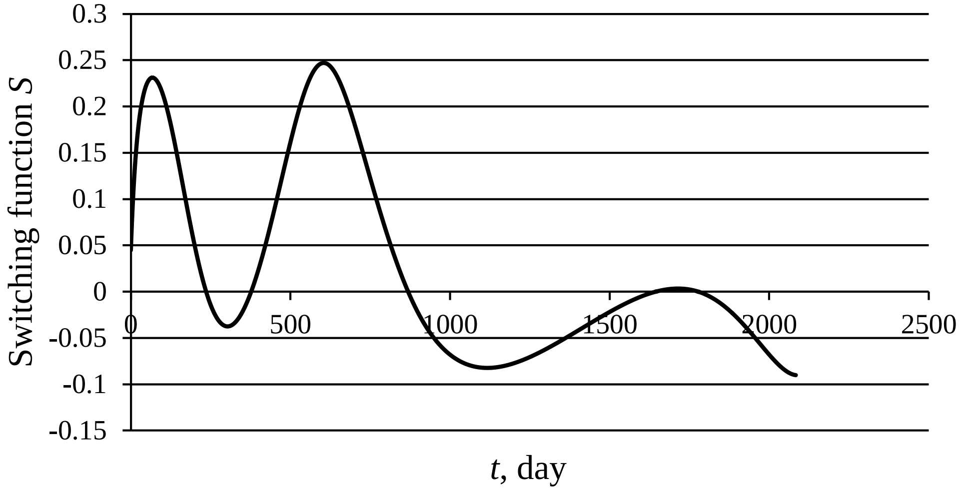
radius	6 671 km
inclination	51.6 deg
ΔV high-thrust	3 150 m/s

Case Sun-Jupiter: Heliocentric segment



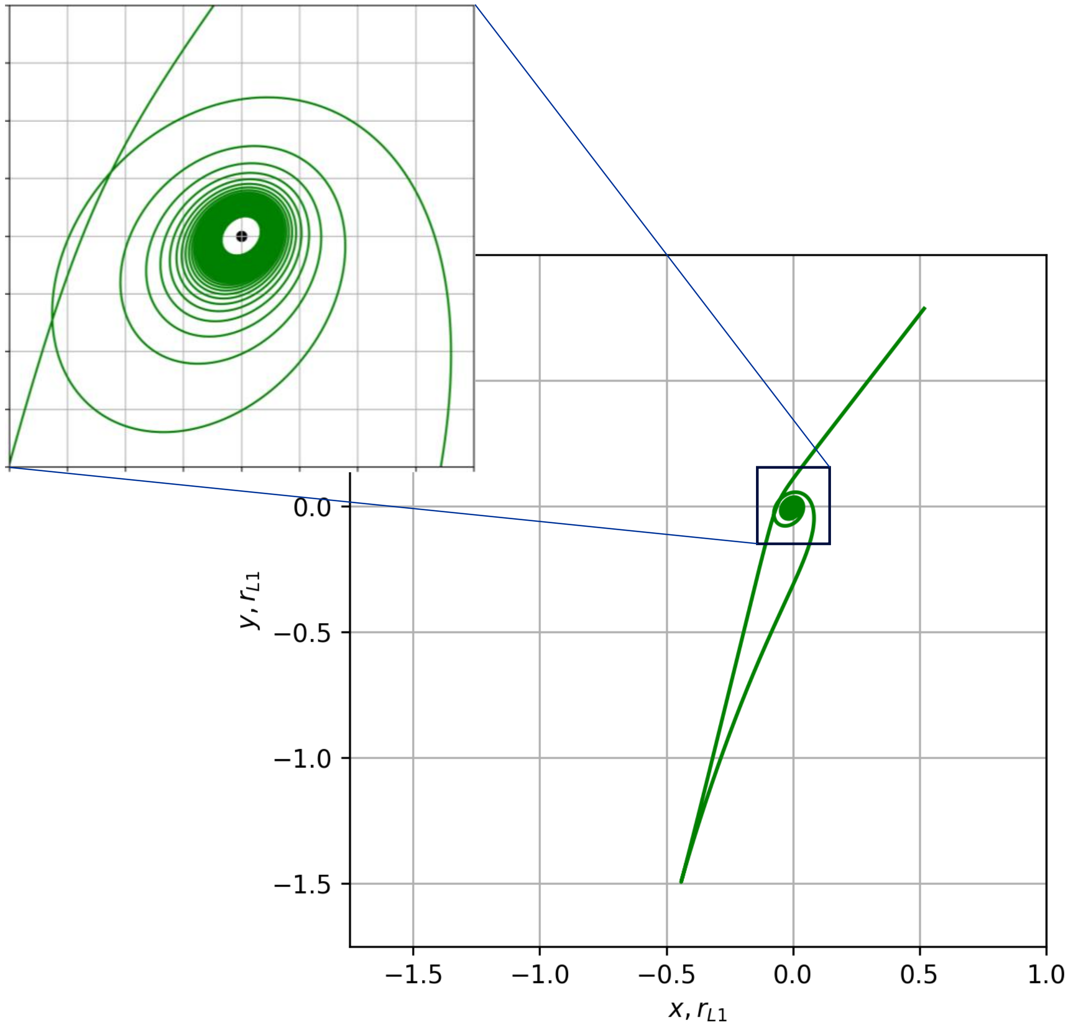
Initial mass SC: 10000 kg
- thrust 1.5 N,
- specific impulse 3000 s



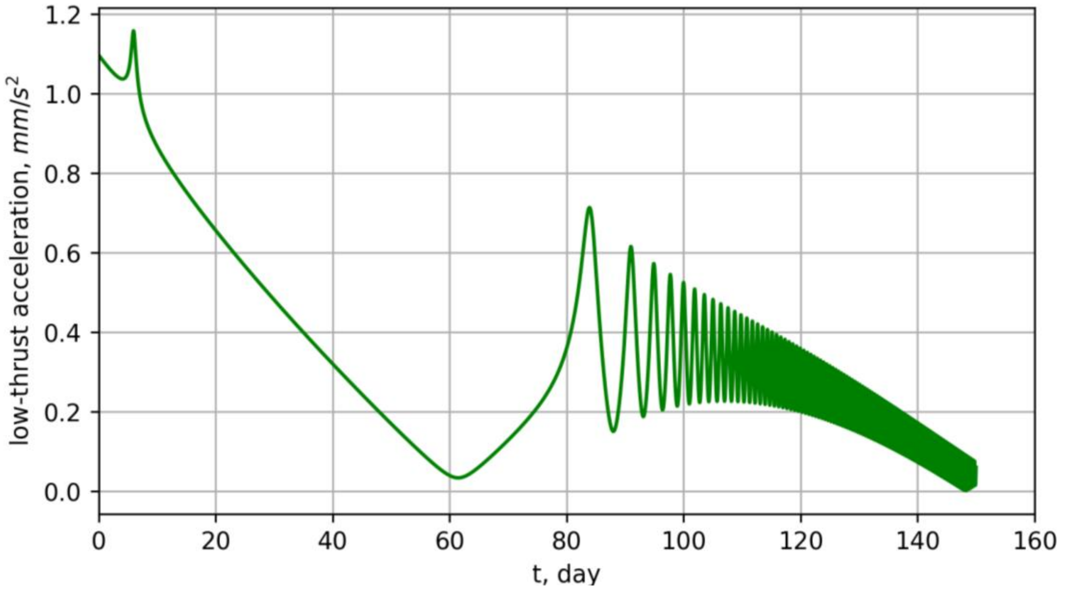


Final mass SC : 6207.027 kg

Case Sun-Mars: Flight near the Mars (Mars L1 – Phobos)



Trajectory Mars L1 - Phobos



Low-thrust acceleration

150 day, 100 rev (Mars L1 - Phobos)

J_{LP} : 1.2680 m^2/s^3

ΔV : 4 592 m/s

Conclusions / Pro et Contra

The proposed approach makes it possible to abandon the zero-sphere of influence model in the analysis of interplanetary missions and design a trajectory using the ephemeris four-bodies problem, to obtain a continuous trajectory in all segments (planetocentric and heliocentric) and appropriate optimal control.

The presented approach has been tested on examples of calculating low-energy transfers to Moon, Jupiter and Martian moons. Has shown its effectiveness. The results obtained are compared with solutions within the framework of the zero-sphere of influence model corresponding to the "classical" schemes of interplanetary flight. As a result, savings of 15-25% of propellant mass are shown when using a low-energy transfer.

- *Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Designing Low-energy low-thrust flight to the Moon on a temporary capture trajectory //* ***Cosmic Research, Vol. 61, No. 5 (2023) 380-393***
- *Ivanyukhin A.V., Ivashkin V.V., Petukhov V.G., Sung Wook Yoon. Low-Energy Lunar Transfer Design Using High-And Low-Thrust on Ballistic Capture Trajectories //* ***Proceedings of the International Astronautical Congress, Paper IAC-23.C1.9.7, IAF Astrodynamics Symposium, 2024, Vol. 2, pp. 896-905.***
- *Sung Wook Yoon, Petukhov V.G., Ivanyukhin A.V., An approach for end-to-end optimization of low-thrust interplanetary trajectories using collinear libration points //* ***Acta Astronautica, 2024, V. 221, pp. 12-25***

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Thanks for your attention